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O.R. Applications

Use of goal programming and integer programming for water quality management—A case study of Gaza Strip

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Abstract

This paper describes a project dealing with achieving an optimum mix of water from different underground wells, each having different amounts of nitrates and chlorides. The amounts of chlorides and nitrates in each of the wells may be higher or lower than the World Health Organization (WHO) standards. Therefore, the optimum mix would be the one that meets WHO standard which is 250 mg/l for chlorides and 50 mg/l for nitrates. A goal programming model was developed to identify the combination of wells along with the amounts of water from each well that upon mixing would result in minimizing the deviation of the amounts of chlorides and nitrates from the standards set by WHO. The output of the goal programming model along with the coordinates of the wells identified above was then used for a second model that determines the locations of the mixing points "reservoirs" in such a way that minimizes the total weighted distances from the corresponding wells. Finally, an easy-to-use pumping schedule was developed using integer programming. Results indicate that for the case study, there exist several optima which make it easier for the decision maker to consider other intangible factors if there are any.

Keywords: Goal programming; Water quality; Nitrates; Chlorides; Gaza

1. Introduction

Gaza Strip is a narrow strip along the Mediterranean sea as shown in Fig. 1. It is around 40 km long while its width varies from 6.4 to 9.6 km. Its area is around 365 km^2 and has a population around 1.1 million. In general, Gaza Strip is known to have serious water problems ranging from quality problems to expected shortage of water supplies not only for agricultural and industrial usage but also for drinking. It is to be noted here that groundwater is the only significant source for water to meet the demand in Gaza Strip. The common two problems with drinking

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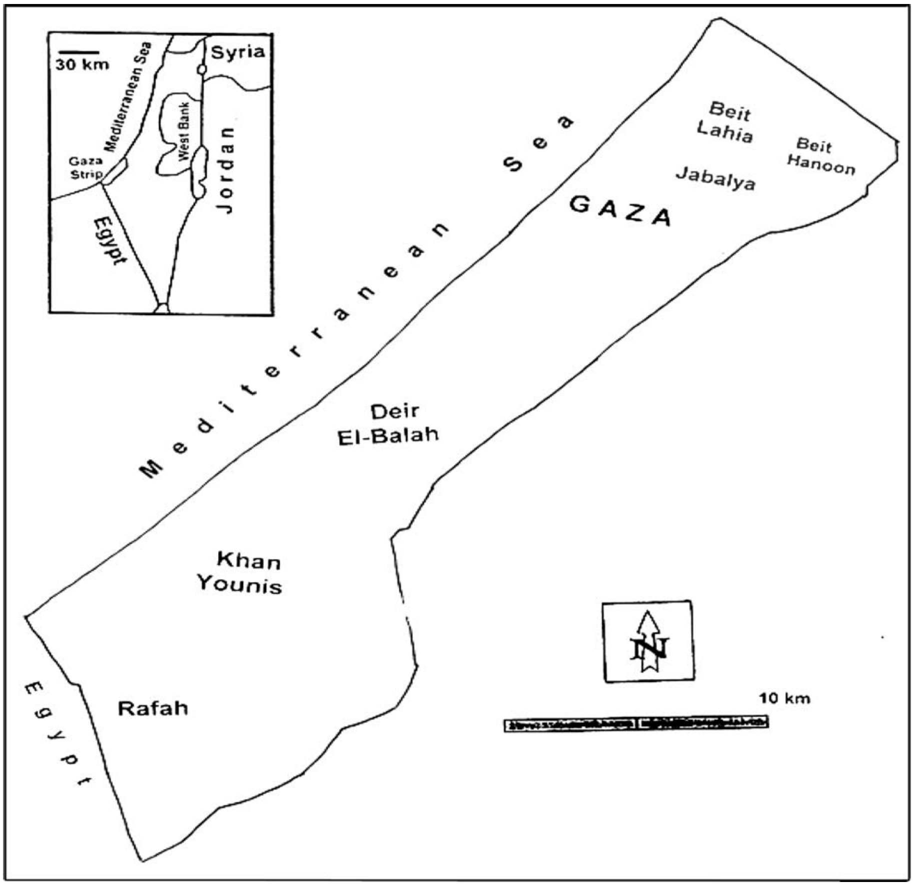

Fig. 1. Location of Gaza Strip.

water quality in Gaza Strip are the presence of high amounts of both nitrates and chlorides. These two problems are mainly attributed to waste water, agricultural fertilizers, pesticides and industrial pollutants. These pollutants may directly penetrate to the groundwater reservoir through pores in rocks or indirectly by decomposition which increases the salts in the wells and thus deteriorates water quality.

In some wells, the level of nitrates in drinking water may exceed 300 mg/l which is 6 times that allowed by (World Health Organization) WHO, while the level of chlorides may exceed the WHO standards by 4–5 times. According to the literature, nitrates level in excess of 150 mg/l poses an extreme risk to infants' health in the form of blue baby syndrome (methaemoglobinaemia) [8].

Moreover, high nitrates may have carcinogenic effects for adults [1]. While the presence of chlorides may not be as harmful as that of nitrates, the salinity it causes, makes the water unacceptable for drinking. Therefore, low level of chlorides is critical for customer satisfaction. In general, when people in Gaza talk about "bad water", they mean high salinity water. Since the levels of nitrates and chlorides are high in some wells and low in others, the objective of this research is to achieve an optimum water mixing strategy in such a way that the amount of chlorides and nitrates are consistent with those approved by WHO. In other words, mixing certain proportions of water from certain wells with the result that the obtained mixture satisfies the WHO standards for both chlorides and nitrates, wells capacity, and customers demand at the least cost. Using water mixing would outperform desalination which is known to reject up to 60% of the input water amount as brine water and this would result in serious environmental problems as cited by [2]. Another draw back of desalination is the high costs involved.

Most of the research performed in this area is of a qualitative nature or at best descriptive and still tends to be more of a data-gathering type of research. This research is an attempt to quantify and model the data that is scattered in the literature to come up with tangible, economically feasible and easily applicable solutions to some of the problems related to drinking water quality in Gaza city. Some studies have focused only on chlorides alone without considering the nitrates. Reports of harmful effects for human health have been reported only for nitrates. Therefore, reducing nitrates level below the specifications of WHO may yield "healthy" water. On the other hand, high chlorides level may render the water undrinkable due to its salinity. In other words, focusing on chlorides only would not solve the problem since people would rather drink non-salty water than water with high level of nitrates which is proved to be dangerous in terms of health. This research considers both nitrates and chlorides using goal programming, which provides a way of striving toward more than one objective function simultaneously. It seeks to establish a specific numeric goal for each of the objectives, and then seeks for a solution that minimizes the weighted sum of deviations of objective functions from their corresponding goals [4]. The goal programming approach is used to seek the amounts of waters from different wells to be mixed in such a way that the deviations from the objective functions are minimized. The use of goal programming or multi-objective programming is very common when dealing with environmental issues where there are normally more than one objective. The following section shows the formulation of the problem as a goal programming problem.

2. Goal programming formulation

Assuming that we have "n" wells, the amount of chlorides in the ith well is "CL_i" mg/l and the amount of nitrates is "N_i" mg/l. Moreover, the ith well can discharge the amount of S_i (m^3/year) at a given operational level. Since the levels of nitrates and chlorides are high in some wells and low in others, it is known that mixing certain proportions from certain wells will achieve the required objective. It is also assumed that there are "m" reservoirs (destinations) each of which can accommodate a certain amount of water that satisfies the demand of the specific region. Let D_j represent the demand required for reservoir "j". Moreover, X_{ij} is the amount of water (m^3/year) discharged from well "i" to reservoir "j". The maximum allowed amount of chlorides per liter (WHO standard) is $\mathrm{CL}_{\max}$, while the maximum allowed amount of nitrates according to WHO standards is $\mathrm{N}_{\max}$.

Therefore, the constraints can be grouped as follows:

- Well capacity constraints

$$\sum_{j=1}^{j=m} X_{ij} \leqslant \mathrm{S}_i \quad (i = 1, \ldots, n). \tag{1}$$

- Reservoirs capacity and demand constraints

$$\sum_{i=1}^{i=n} X_{ij} \geqslant D_j \quad (j = 1, \ldots, m). \tag{2}$$

- Chlorides balance constraints

$$\sum_{i=1}^{i=n} X_{ij}\mathrm{CL}_i - \sum_{i=1}^{i=n} X_{ij}\mathrm{CL}_{\max} - Y_{j\mathrm{CL}}^{+} + Y_{j\mathrm{CL}}^{-} \leqslant 0 \quad (j = 1, \ldots, m). \tag{3}$$

- Nitrates balance constraints

$$\sum_{i=1}^{i=n} X_{ij}\mathrm{N}_i - \sum_{i=1}^{i=n} X_{ij}\mathrm{N}_{\max} - Y_{jN}^{+} + Y_{jN}^{-} \leqslant 0 \quad (j = 1, \ldots, m). \tag{4}$$

All variables are $>= 0$,

where $Y^{+}_{j\mathrm{CL}}$ is the excess of chlorides from the WHO standard in reservoir "j"; $Y^{-}_{j\mathrm{CL}}$ is the amount by which chlorides in reservoir "j" is below the WHO standard; $Y^{+}_{j\mathrm{N}}$ is the excess of nitrates from the WHO standard in reservoir "j"; $Y^{-}_{j\mathrm{N}}$ is the amount by which nitrates in reservoir "j" is below the WHO standard.

Finally the objective function can be written as follows:

$$\mathrm{Min}\, z_1 = \sum_{j=1}^{j=m} Y^{+}_{jN} + \sum_{j=1}^{j=m} Y^{+}_{j\mathrm{CL}}.$$

The set of constraints in Eq. (1) guarantees that the total amount of the discharged water from certain wells does not exceed the capacity these wells. While the constraints in Eq. (2) require that the amount of water supplied to a given reservoir from given wells does not exceed the capacity of that reservoir. Eqs. (3) and (4) respectively represent the chlorides and nitrates balance before and after mixing.

The assumption in the (chlorides/nitrates) balance constraints is that there are no reactions due to water mixing which may decrease the amounts of both elements under study. Therefore, this assumption, though approximate, guarantees that the levels of nitrates and chlorides can always be equal to or less than the amount required by WHO. Moreover, it is assumed that the system experiences no loss of nitrates as shown by preliminary experiments. In other words, the results will always be on the conservative side. Solution of the above model results in obtaining combination of wells and the amounts of water from each well. Due to the fact that the existing reservoirs are pretty old, it was suggested that new reservoirs be built. Thus, it was decided to take the extra mile to find the location of these reservoirs as will be discussed in the next section.

3. Reservoirs locations

The location of the reservoirs is found using the center of gravity method according to the following equation using rectilinear distance, which is a reasonable assumption given the topography of Gaza city.

$$\mathrm{Min}\, f(x) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(X_j, X_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(X_j, P_k),$$

where $X_j = (x_j, y_j)$ location of new facility j, $j = 1, 2, \ldots, n$; $P = (a_i, b_i)$ location of existing facility i, $i = 1, 2, \ldots, m$; V_{jk} cost per unit distance travel between new facilities j and k; W_{jk} cost per unit distance travel between j and i; $d(X_j, X_k)$ distance between j and k; $d(X_j, P_k)$ distance between j and i with rectilinear distance

$$d(X_j, X_k) = |x_j - x_k| + |y_j - y_k|,$$
$$d(X_j, P_k) = |x_j - a_i| + |y_j - b_i|.$$

4. Operations scheduling

Scheduling of the wells pumping into the given reservoirs was done based on the fact that wells pumping into the same reservoir operate simultaneously in order to ensure the intended mixing (a well-mixed water). Moreover, a constraint was added due to the limitations that each well can only pump into a single reservoir at a single time.

The formulation uses three equally sized reservoirs and assumes that pumping can take place in two time periods during the 24 hours. In other words, pumping can occur in time period 1 or 2.

5. Application

Gaza city municipal wells were chosen for the application. Gaza city has some 20 municipal wells that supply the population with the drinking water. Fig. 2 shows the map of the wells distribution. Table 1 shows these wells, the discharge capacity for each well, and chemical analysis of the amounts of nitrates and chlorides in each. The consumption of drinking water throughout the area is estimated to be 4.2×10^6 m^3/year [6]. What makes the model easily applicable in Gaza city is the fact that most houses have two water mains one for drinking

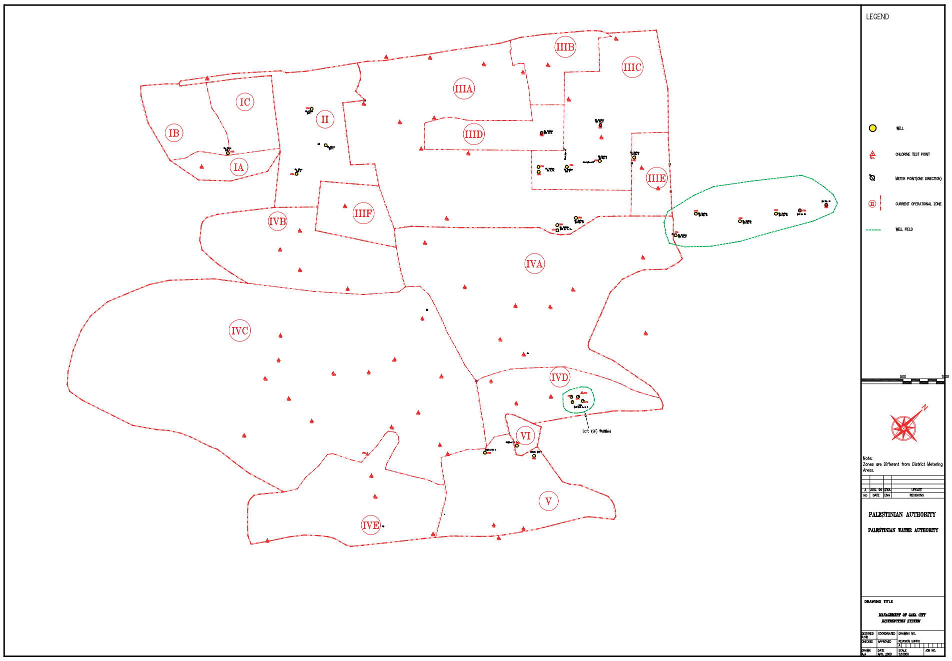

Fig. 2. Wells distribution in Gaza city.

Table 1
Output and chemical analysis of wells under study [7]

No.	Well name	Q (m^3/hour)	Chlorides (mg/l)	Nitrates (mg/l)
1	Sheikh Radwan 1	180	273	135
2	Sheikh Radwan 1A	180	245	140
3	Sheikh Radwan 3	150	1015	135
4	Sheikh Radwan 4	180	1085	90
5	Sheikh Radwan 7	180	553	175
6	Sheikh Radwan 7A	180	485	115
7	Sheikh Radwan 8	150	133	80
8	Sheikh Radwan 9	190	133	90
9	Sheikh Radwan 10	190	90	60
10	Sheikh Radwan 11	190	110	80
11	Sheikh Radwan 12	180	110	45
12	Sheikh Radwan 13	180	460	200
13	Sheikh Radwan 15	180	110	45
14	Sheikh Radwan 16	180	110	45
15	Sheikh Ejleen 1	150	810	110
16	Sheikh Ejleen 2	120	440	80
17	Saffa well 1	200	600	225
18	Saffa well 2	150	400	125
19	Saffa well 3	100	740	215
20	Saffa well 4	180	590	110

and the other is for other purposes. Drinking water is supplied to houses using a certain schedule for each area or community. It is to be noted here that the size of each reservoir is large enough that it will meet the demand with no need to take demand fluctuations into account. In other words, the reservoirs act like a buffer against the fluctuations in demand.

6. Results and discussion

The following sections include the obtained results and a discussion of the implications of these results.

6.1. Amounts and quality of mixed water in reservoirs

The solution to this problem was generated using LINDO version 6 [5]. Different operating hours for the wells were used. Twenty hours per day and 18 hour/day were found to achieve "zero" deviations from the WHO standard (50 mg/l nitrates and 250 mg/l chlorides [8]). While 12 hour/day operation was found to give 240 mg/l of nitrates in reservoir 1 which means an excess of 190 mg/l from the WHO standard, while water in reservoirs 2 and 3 exactly meet the WHO specifications. As for the chlorides levels in reservoirs 2 and 3, they were found to be 105 mg/l and 103 mg/l respectively. Both of these values are below the WHO specification for chlorides. It is noted that different operating hours affect the obtained quality. This can be attributed to the fact that different operating hours would result in different combinations of wells since wells have different concentrations of chlorides and nitrates and limited capacities.

Therefore, and based on practical considerations, the 18 hour/day was used as the solution recommended, however, as will be shown later, only one well will operate for 18 hour. Then, looking at the well distribution it was found that some of the furthest wells could be eliminated while still achieving the "zero" deviation.

As for the recommended solution, the excess of nitrates from the WHO specifications in each of the three reservoirs is "zero". In other words, the nitrates level exactly meets the WHO specifications. While the level of chlorides in reservoir 1 is 108 mg/l below the WHO standard and 32 mg/l below the standard in reservoir 2 and 3. Table 2 shows the obtained quality of the mixed water in each of the reservoirs. It is clear from the table

Table 2
Resulting amounts of nitrates and chlorides in reservoirs in mg/l

	Reservoir 1	Reservoir 2	Reservoir 3
Nitrates	50	50	50
Chlorides	142	218	218

Table 4
Relative coordinates of location of wells and reservoirs

Well no.	*X*	*Y*
2	143,213	3941
4	143,943	4070
9	145,089	3443
11	146,046	3443
13	146,329	3482
14	146,644	3548
Reservoir 1	146,046	3443
Reservoir 2	146,644	3548
Reservoir 3	146,329	3482

Table 3
Optimum combination of wells and quantities of water supplied to reservoirs in m^3/year

Well no.	Reservoir 1	Reservoir 2	Reservoir 3
2	23,546		
4		155,556	155,556
9	317,543		
11	1,058,911		123,689
13		61,845	1,120,756
14		1,182,600	
	Total = 1.4×10^6	Total = 1.4×10^6	Total = 1.4×10^6

Table 5
A feasible operation schedule in hour/day

Well	Period 1			Period 2		
	Reservoir 1	Reservoir 2	Reservoir 3	Reservoir 1	Reservoir 2	Reservoir 3
2	0.36					
4		2.87				2.87
9	4.58					
11	16.12					1.88
13		0.95				17.05
14		18				

that the WHO specifications are met for the three reservoirs. The obtained quality in each of the reservoirs is expressed in terms of the maximum chlorides or nitrates concentration rather than the distribution. Expressing the quality in terms of the maximum shows that the obtained quality meets the standards. Therefore, any value lower than that would automatically satisfy the standards.

Table 3 shows the allocation of wells to the given reservoirs along with the amounts of water to be pumped to each of these reservoirs. The data obtained was used to locate the three reservoirs that would meet the required drinking water to the population in that area. A forecasting model was developed to predict both the amounts of chlorides and nitrates in each of the wells after being used for a certain number of years. The results indicate that the obtained combination may work for 3 more years by simply adding another well of the unused [3].

6.2. Location of the reservoirs

In order to minimize the transportation cost, the location of the three reservoirs was calculated based on the supplying wells, their coordinates and the amount of water each well supplies to a given reservoir using the center of gravity method. Table 4 shows the relative locations of each of the wells and the reservoirs based on the Euclidean distance. As can be seen from the table, it turns out that reservoir 1 should be located as close as possible to well 11. While reservoir 2 has to be placed as close as possible to well 14. Finally, reservoir 3 has to be located as close to well 13 as possible.

6.3. Operations scheduling

Once the optimum combination of wells and reservoirs was obtained, some operational constraints have been added using integer programming so that wells assigned to a given reservoir operate simultaneously in order to avoid the problem of not having a well-mixed water. Moreover, a constraint was added such that a well does not supply more than one reservoir at the same time.

Table 5 shows the pumping schedule of each of the given wells to each of the reservoirs. It is noticed from the table that pumping into reservoirs 1 and 2 takes place during time period 1. For reservoir 1, well 2 operates for 0.36 hour, well 9 operates for 4.58 hours while the remaining amount comes from well 11 which should run for around 16 hours.

7. Conclusions

A goal programming model was developed for the management of water quality and quantity for Gaza city in Gaza Strip. The model had three objectives: (1) minimize the deviation of chlorides and nitrates from the WHO standard by mixing water from different wells while satisfying customer needs and well capacities, (2) find the optimal location of a number of reservoirs for the mixing phase and (3) an operational schedule was designed so that it can be easily used by the operator. The solution shows that it is feasible to supply the customers with water that is both acceptable for drinking (due to presence of chlorides) and simultaneously reduces the risks associated with the high levels of nitrates. The model can be extended to include more than two operation periods. It can also be applied to other areas in the Gaza Strip and take into account other pollutants like fluoride and boron for example. Finally, wells with levels of nitrates and chlorides that are highly intolerable can be used for drinking purposes upon mixing. This technique offers a cheaper and more environment-friendly solution to the water quality problem in Gaza Strip than desalination.

References

[1] K.P. Cantor, Drinking water and cancer, Cancer Causes Control 8 (3) (1997) 292–308.

[2] H.A. El-Nakhal, Alternatives to tap water: A case study of the Gaza Strip Palestine, Environmental Geology 46 (6–7) (2004) 851–856.

[3] J. Hamad, Sea water intrusion in Gaza Strip, Unpublished Report, Department of Civil Engineering, The Islamic University of Gaza, 2004.

[4] F.S. Hillier, G.J. Lieberman, Introduction to Operations Research, seventh ed., McGraw-Hill, NY, 2001.

[5] L. Scharge, Optimization Modeling with LINDO, fifth ed., Brooks/Cole Publishing Co., Pacific Grove, CA, 1997.

[6] MENA, Palestine Environmental Strategy, Palestinian Water Authority, Ministry of Environmental Affairs, 1999.

[7] Palestinian Water Authority (PWA)/USAID, Coastal Aquifer Management Program (CAMP), Integrated Aquifer Management Plan (Task-3), Gaza, 2002.

[8] World Health Organization, Guidelines for drinking water quality, second ed., 2 Health Criteria and Other Supporting Information, WHO, Geneva, 1996.