Journal of Al - Azhar University - Gaza

Natural Sciences

This special issue is released on the occasion of the International Conference on Basic and Aplied Sciences (ICBAS2010)
10-12 October 2010

A Refereed Scientific Journal

Published by
Deanship of Postgraduate Studies and Scientific Research
Al - Azhar University - Gaza
Palestine

ISSN 1810-6366

Volume: 12, ICBAS Special Issue

Mesoscopic Transport and Persistent Current in the Aharonov-Bohm Rings

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Abstract: *The electronic transport of a non-interacting quantum rings serially connected to a quantum wire is investigated via the transfer matrix method. We discuss in detail some features of the band structure in multiple rings configuration in the presence of a magnetic field. Moreover, for a uniform quantum ring, the persistent current is found to exhibit a periodic band structure. The effect of magnetic field is also studied. This field is shown to shift the electronic spectrum and damp the amplitude of the persistent current density.*

Key words: *quantum transport, Aharonov-Bohn rings, persistent current.*

INTRODUCTION

In recent years, electronic transport properties of mesoscopic systems in particular, the Aharonov-Bohn rings have been extensively investigated both theoretically and experimentally [1-4]. By virtue of highly developed nanotechnology, some interesting phenomena based on quantum coherence have been observed. One of the interesting mesoscopic phenomena predicted by theory [5] and verified experimentally [6] is that case when the persistent current is flowing in a ring of mesoscopic size with a piercing magnetic flux through the ring. An additional phase evolution is involved to satisfy the invariance when a magnetic flux passing through the ring. This additional phase evolution is responsible for the persistent current in a closed ring. The well-known Aharonov-Bohm (AB) effect is the interference due to the phase difference. The physical basis underlying the persistent current in the mesoscopic ring with a piercing flux is the difference in phase evolution along different paths of the ring. The piercing flux and the mesoscopic ring threaded by this flux are called the AB flux and the AB ring, respectively. A great deal of research has been made on the single ring [7,8] and coupled ring [9] structures. These studies show us that the persistent current can be generated not only in isolated rings but also in the rings with external leads connected to electron reservoirs. In this paper, a transfer matrix approach is presented for the study of electronic transport and persistent current in a non-interacting quantum rings serially connected to a quantum wire in the presence of a uniform magnetic field. The effect of the geometrical features on the electronic transmission is discussed.

This paper is organized as follows. In section 2 we examine the behavior of the transmission in a single loop and serially connected ones. We discuss a peculiar behavior of the persistent current in the AB ring in section 3. We give conclusions of this work in section 4.

TRANSMISSION IN PERIODIC RINGS

For our purposes, we will discuss the transmission in serial arrangement of one dimensional (1D) rings in the presence of a magnetic field perpendicular to the ring plane as shown in Fig.1

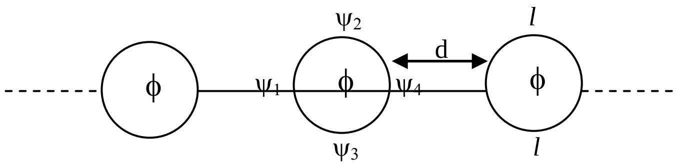

Fig.1. Serial ring structure consisting of *N* rings.

The two arms of the rings are of equal length *l* each and the distance between the rings is taken to be *d*. Schrödinger's wave equation of a ring in a magnetic field is given by:

$$\left[\frac{1}{2m^*}\left(\hbar\nabla^2-\frac{e}{c}\vec{A}\right)^2+V(\vec{r})\right]\psi(\vec{r})=E\psi(\vec{r}) \tag{1}$$

where $\hat{A}=\nabla\times\hat{B}$ is the vector potential of the magnetic field **B**. The geometry of the ring allows to relate the vector potential to the magnetic flux Φ. According to the Gauss's theorem, the vector potential $\hat{A}$ is along the ring direction and its magnitude is:

$$A=\frac{\Phi}{L} \quad , \qquad \Phi=\oint \vec{B}.d\vec{s} \;=\; BS \qquad (2)$$

where S is the ring area and L is its circumference. The Schrödinger's wave equation of the 1D system can be rewritten replacing A by Φ/L, that is:

$$\left[\frac{\hbar^2}{2m^*}\left(\frac{1}{i}\frac{d}{dx}-\frac{\alpha}{L}\right)^2+V(x)\right]\psi(x)=E\psi(x) \qquad (3)$$

We have introduced the reduced flux α as:

$$\alpha=\frac{e\phi}{\hbar c}=2\pi\frac{\phi}{hc/e}=2\pi\frac{\phi}{\phi_o} \qquad (4)$$

where $\phi_0 = hc/e$ is the magnetic flux quantum.

In our system, we consider only the geometrical effects on the electron transport so that $V(x)=0$. The effect of a magnetic field on the wave function ψ (x) can be interpreted as a gauge transformation from zero flux ($\alpha = 0$) to non-zero flux ($\alpha \neq 0$):

$$Ho=\frac{\hbar^2}{2m^*}\left(\frac{1}{i}\frac{d}{dx}\right)^2 \Rightarrow H_\alpha=\frac{\hbar^2}{2m^*}\left(\frac{1}{i}\frac{d}{dx}-\frac{\alpha}{l}\right)^2 \qquad (5)$$

The absolute value of the vector potential is proportional to the magnetic flux, $A=\alpha/l$. Therefore, the global phase factor appearing in the wave function due to the gauge transformation contains the magnetic flux α as in the form:

$$\psi(x)\rightarrow e^{i\chi(x)}\psi(x)=e^{i\alpha\frac{x}{l}}\psi(x) \qquad (6)$$

The wave function $\psi(x)$ is still a plane wave and the net effect of a magnetic field appears as a change in the phase of the wave function. For instance, an electron traveling from the left lead to the upper arm acquires a phase factor $e^{i\pi\alpha}$ and one along the lower arm acquires $e^{-i\pi\alpha}$.

In the local coordinate system, the wave functions in the two arms of the j[th] loop and its connected leads can be written as;

$$\psi_1=a_1e^{ikx}+b_1e^{-ikx} \qquad (7.a)$$

for the left lead

$$\psi_2=a_2e^{ikx}+b_2e^{-ikx-i\pi\alpha} \qquad (7.b)$$

for the upper arm

$$\psi_3=a_3e^{ikx}+b_3e^{-ikx+i\pi\alpha} \qquad (7.c)$$

for the lower arm, and

$$\psi_4=a_4e^{ikx}+b_4e^{-ikx} \qquad (7.d)$$

for the right lead, where the wave number is $k=(2m^*E/\hbar^2)^{\frac{1}{2}}$.

The boundary conditions, conservation of current density and continuity of the wave functions, are to be imposed on the two junctions; we have a set of equations:

$$a_1e^{ikd}+b_1e^{-ikd}=a_2+b_2e^{-i\pi\alpha} \qquad (8.a)$$

$$a_1e^{ikd}+b_1e^{-ikd}=a_3+b_3e^{i\pi\alpha} \qquad (8.b)$$

$$a_2e^{ikl+i\pi\alpha}+b_2e^{-ikl}=a_4e^{ikl}+b_4e^{-ikl} \qquad (8.c)$$

$$a_3e^{ikl-i\pi\alpha}+b_3e^{-ikl}=a_4e^{ikl}+b_4e^{-ikl} \qquad (8.d)$$

and the conservation of the current gives:

$$a_1e^{ikl}-b_1e^{-ikl}=a_2-b_2e^{-i\pi\alpha}+a_3-b_3e^{i\pi\alpha} \qquad (8.e)$$

and

$$a_2e^{ikl+i\pi\alpha}-b_2e^{-ikl}+a_3e^{ikl-i\pi\alpha}-b_3e^{-ikl}=a_4e^{ikl}-b_4e^{-ikl} \qquad (8.f)$$

In order to obtain the transmission spectrum for the electron system, we have used the transfer matrix method which relates the coefficients of the wave function at one end to those at the other end.

The transfer matrix through the *j-th* loop is given through the relation:

$$M^{(j)}=\begin{bmatrix}1/t^* & r/t\\ r^*/t^* & 1/t\end{bmatrix} \qquad (9)$$

Where j = 1, 2, 3, …N, t and r are the complex amplitudes of the transmission and reflection while the asterisk (*) denotes the complex conjugation. The total transfer matrix for the case of N loops arranged in series can be expressed as:

$$M=M^{(N)}S\,M^{(N-1)}S...M^{(2)}SM^{(1)} \qquad (10)$$

Where,

$$S=\begin{pmatrix}e^{(ikd)} & 0\\ 0 & e^{(-ikd)}\end{pmatrix} \qquad (11.a)$$

and

$$M^{(j)} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \quad (11.b)$$

where the matrix elements are given by:

$$M_{11} = \frac{2\sin 2kl - (1-5\cos^2 kl + 4\cos^2 \pi\alpha)i}{4\sin kl \cos \pi\alpha} \quad (12.a)$$

$$M_{12} = \frac{(1+3\cos^2 kl - 4\cos^2 \pi\alpha)i}{4\sin kl \cos \pi\alpha} \quad (12.b)$$

$$M_{21} = -\frac{(1+3\cos^2 kl - 4\cos^2 \pi\alpha)i}{4\sin kl \cos \pi\alpha} \quad (12.c)$$

and

$$M_{22} = \frac{2\sin 2kl + (1-5\cos^2 kl + 4\cos^2 \pi\alpha)i}{4\sin kl \cos \pi\alpha}. \quad (12.d)$$

On substituting for $\alpha = 0$ we end typically with the common matrix elements in absence of the magnetic field[10].

Now, all the loops are identical, that is:

$$M^{(N)} = M^{(N-1)} = \ldots = M^{(1)} \quad (13)$$

The total transfer matrix that represents the electron wave propagation through the entire device is just the product of the transfer matrices of the loops in order.

Thus we obtain:

$$M^{tot} = \begin{bmatrix} 1/t_N^* & r_N/t_N \\ r_N^*/t_N & 1/t_N \end{bmatrix} \quad (14)$$

or:

$$M^{tot} = S^{-1}M^{(N)}\left(\prod_{j=1}^{N} SM^{(j)}\right) \quad (15)$$

From this we can find the total transmission and reflection amplitudes across the periodic structure of *N* rings.

The transmissions amplitude of the electron is related to the total transfer matrix M^{tot} as [11]:

$$t = 1/M_{(2,2)}^{tot} \quad (16)$$

so that the transmission coefficient is given by:

$$T = \frac{1}{|t|^2} \quad (17)$$

These transmission coefficients are periodic in both α and *kl* [12] with period 1 and π respectively. Fig.2. illustrates the lines of total transmission of reflection along with the singular points of the coefficients of Eq.14 for a single ring.

We first find the conditions for total reflection (*T*=0) and transmission (*T*=1) in the ring geometry considered in this work. Total reflection occurs when the electron wave functions in the upper and lower arms are completely canceled at the branch point of the ring due to perfect interference, which occurs when the phase difference between the wave functions of the upper and lower arms of the ring is $(2n+1)\pi$, where $n = 0, 1, 2, \ldots$ This condition is satisfied when the flux α has half – integral numbers irrespective of the value of *kl*. Since half-integral number of α is a condition for zero transmission.

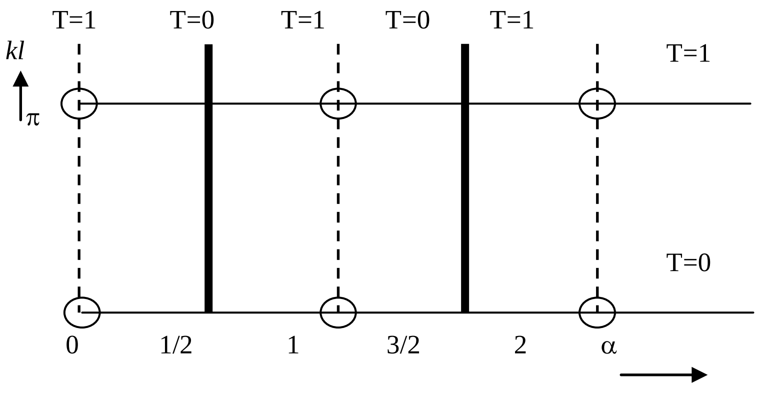

Fig.2. Plots of singular points of transmission coefficients. Thick vertical solid lines represents *T*=0, while thin horizontal lines represent flux independent *T*=0. Open circles are the points of *T*=1 and the vertical dotted lines are just guides.

Another condition called the total transmission exists when there is no scattering or reflection at any branch points. Scattering occurs when the wave functions in the upper and lower arms of the ring are not matched completely with the ring geometry. This mismatching [13] stems from both the wave vector *k* of the electron, and the amount of piercing flux α Both of these two considerations can change the phase of the electron wave function. Therefore, total transmission occurs only when the following two conditions are satisfied simultaneously: i) the nodes of the wave function should match perfectly with the branch points of the ring, i.e. *kl* should be an integral number multiple of π; ii) the piercing flux α must be an integral number so that the system does not experience the existence of the piercing flux. We denote the points satisfying the above two conditions simultaneously as open circles in Fig.2.

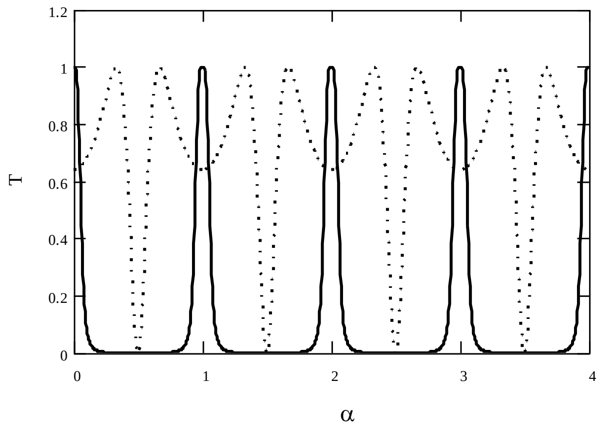

Fig. 3. Transmission coefficient versus α for a single loop. The solid curve corresponds to kl =0.49π and while the dotted curve corresponds to kl=0.01π.

Note that, we do not include in Fig.2. the case of total transmission occurring at non integral values of kl/π and α, such as the peak points giving T=1 shown in Fig.3. where we plot T as a function of α. we did not show this, they are the combined effect of the wave vector and the piercing flux in the phase evolution of the wave function. Both wave vector and piercing flux cause a deviation from perfect matching. An example is shown in Fig.3. If *kl* deviates from an integral multiple of π even when the flux α is an integral number, the transmission coefficient becomes less than unity. Perfect matching in this case occurs at non integral values of α. As a typical case, the transmission coefficients at kl/π =0.01 near resonance and kl/π =0.49 near anti-resonance are shown in Fig.3.

We now consider the evolution of the band structure for the transmission coefficient as a function of kl/π in the serial ring structure. In the presence of the magnetic flux through each ring, each band splits into two except for the lowest one. The band gap appears at the center of each band.

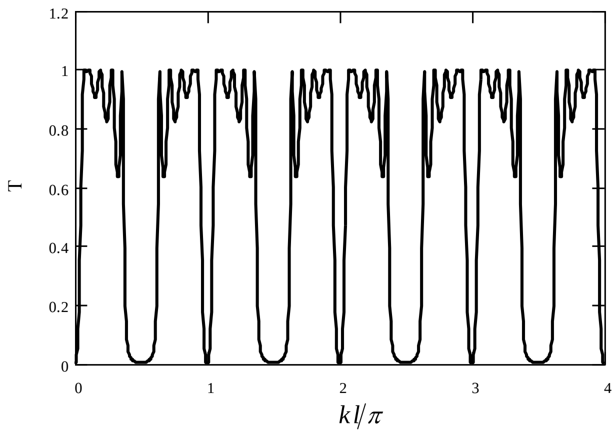

Fig.4 Transmission coefficient versus kl/π for N = 5, AB ring with l/d = 1.0 and magnetic flux α = 0.2/π.

This behavior is shown clearly in Fig.4, as we have taken α =0.2/π and l/d =1.0 for N=5 rings. The band structure evolves continuously as the magnetic flux is changed, the band gap increases continuously with the magnetic flux. In Fig.5 the dashed curve is for N=5, l/d =1.0, and α=0.4/π whereas the solid curve is for N=5 , l/d = 1.0 and α = 0.2/π. The band gap is seen to be wider for the larger magnetic flux. In this way these special band gaps initially keep widening with α up to α=1/π, where the transmission is identically zero for all values of kl/π and then it starts narrowing with α, finally disappearing at α=2/π. If we increase α further, then the same cycle is repeated, with a period of 2π. Each maximum of the unit transmission splits into two and a zero develops in between. This is because the magnetic field breaks the time-reversed symmetry [5]. In fact, depending on the adjustable parameters of the ring structure, we can find a large number of transmission spectra that can be used to operate electronic devices.

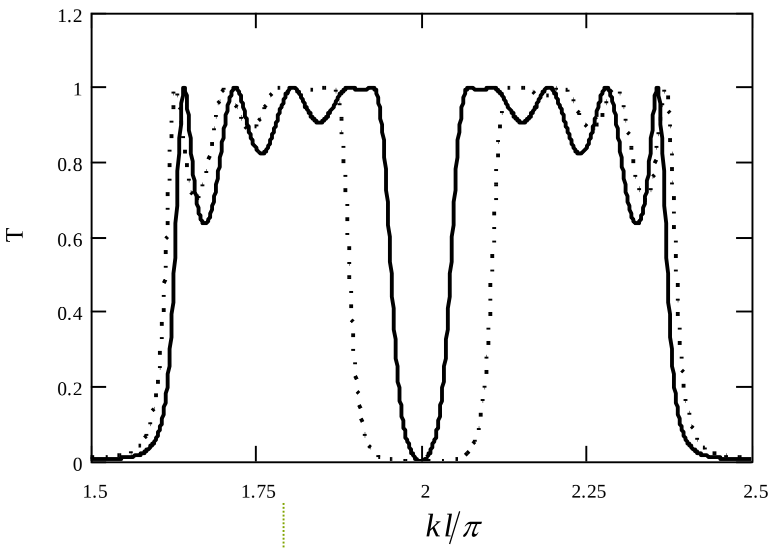

Fig.5. Transmission coefficient versus kl/π for N = 5 , AB ring with l/d = 1.0 and magnetic flux for two values. The solid curve corresponds to α =0.2/π and the dashed curve corresponds to α = 0.4/π

PERSISTENT CURRENT

The persistent currents of AB ring are calculated from the usual formula for current density in the presence of magnetic flux as [14]

$$J_i = \frac{e\hbar}{2mi}(\psi_i^* \nabla \psi_i - \psi_i \nabla \psi_i^* - 2i\frac{\alpha}{l}\psi_i^* \psi_i) \qquad (18)$$

which implies that:

$$I_p^i = \frac{J_i}{e\hbar k/m} \qquad (19)$$

where I_p is the persistent current and (i) denotes the upper lower arms of the ring. For instance, the probability currents can be expressed for the upper arms as:

$$l_{up} = |a_2|^2 - |b_2|^2 \qquad (20.a)$$

and

$$l_{low} = |a_3|^2 - |b_3|^2 \qquad (20.b)$$

for the lower arm,(see Eq.7).

Persistent current is defined by the net circulating current [8] and expressed by:

$$l_p = | l_t/2 - l_c | \qquad (21)$$

where I_t is the transmitting probability current that is just the transmission coefficient T and I_c is the circulating probability current.

Since we may write $|I_{up}| = |I_t/2| + |I_c|$ and $|I_{low}| = |I_c| - |I_t/2|$, the circulating current is expressed by $|I_c| = (|I_{up}| + |I_{low}|)/2$. Therefore, the net circulating current, i.e. the persistent current in the AB ring is given by:

$$I_p = \frac{T - |I_{up}| - |I_{low}|}{2} \qquad (22)$$

Here the current in the upper arm is given by:

$$I_{up} = \frac{T}{2}\left(1 - 2\frac{\tan \pi\alpha}{\tan kl}\right) \qquad (23)$$

while in the lower arm can be obtained by replacing α by -α.

The currents in the upper and lower arms are generally different by symmetry breaking. If the current in one arm is larger than *T*, the current in the other arm must be negative to conserve the total current at the junction, giving the full meaning of the circulating current in the ring.

The types of current flow transmitting from left to right reservoir are important in special *T*=1. For this purpose, we calculate the currents I_{up} and I_{low} at each arm of the ring near $kl = \pi$ as shown in Fig.6.

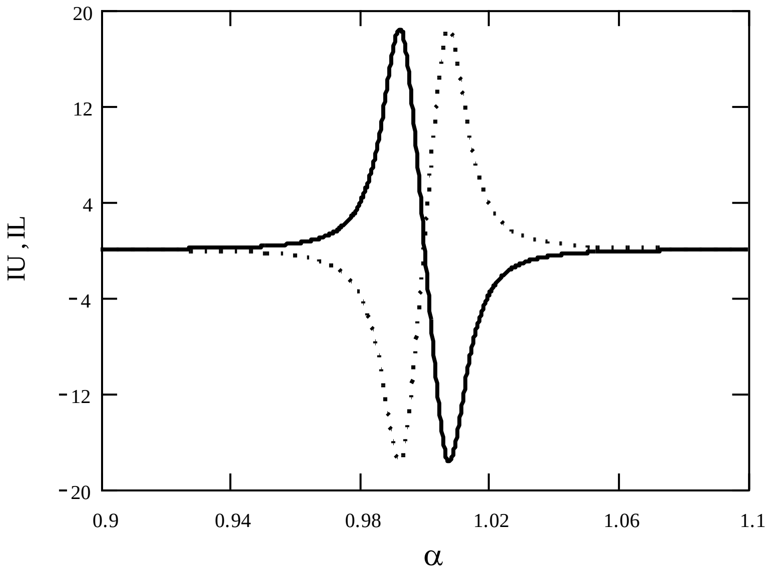

Fig. 6. Current densities in each arm of a single ring at $kl=\pi + 0.01$. The solid curve is plotted for I_{up} and the dotted curve for I_{low}.

The different signs of the current densities in the upper and lower arms indicate the existence of a circulating current. A further notable thing is that the values of coefficients are much larger than unity, which implies that the normalization is not introduced. Therefore, the values indicate relative strength of current. Accordingly, there are two types of current in each arm of the ring [2]. One is the circulating current responsible for the persistent current and the other is the transmitting current from left to right reservoir.

CONCLUSION

We have analyzed the transmission coefficient and the persistent current in the periodic multiple AB rings. Throughout the work, we have restricted ourselves to a single channel free electron device. In this structure, the scattering is solely due to the geometric nature of rings (Ballistic transport). We found that the flux independent total reflection and transmission appear at geometrically matched cases, i.e. $kl = n\pi$ and $kl = (2n+1)\ \pi/2$, respectively.

The evolution of the band structure of transmission, as a function of magnetic flux and a ring number, have been pointed out. The sensitivity of the band structure to the magnetic flux can be utilized for band tailoring and for quantum device operations .

In addition, the characteristics of current flow in the upper and lower arms of each ring flow anti-symmetrically. The persistent currents are similar in each ring because of the similarity of the wave functions in each ring. In the numerical calculations, we have found that the transfer matrix method is more efficient and has more generality than other methods in the literature [13,15].

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ISSN 1810-6366

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