Performance Evaluation of MIMO Spatial Multiplexing Detection Techniques

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***Abstract**:*
Multiple Input Multiple Output (MIMO) multiplexing is a promising technology that can greatly increase the channel capacity without additional spectral resources. The challenge is to design detection algorithms that can recover transmitted signals with acceptable complexity and high performance. In this paper, several MIMO Spatial Multiplexing (SM) detection techniques are introduced and evaluated in terms of BER. Different aspects have been considered and discussed in this evaluation such as; signal to noise ratio, number of transmit and receive antennas. The performance comparisons and graphs have been generated using an optimized simulator. This simulator has been developed using MATLAB®.

Key words: *MIMO, spatial multiplexing, maximum-likelihood detection, linear detection, tree search.*

I. Introduction

During the last decade, the intensive work of researches on Multiple-Input Multiple-Output (MIMO) systems have demonstrated their key role in increasing the channel reliability and improving the spectral efficiency in wireless technology without the need to additional spectral resources (Wolniansky et al., 1998). Recent developments have shown that using spatial multiplexing MIMO systems can increase the capacity substantially without requiring extra-bandwidth or transmit power (Wolniansky et al., 1998), (Foschini and Gans 1998). MIMO technology is thus categorized into two main categories, namely; spatial multiplexing and MIMO diversity schemes. In spatial multiplexing systems, independent data streams are transmitted simultaneously via different transmit antennas. As a consequence, the channel capacity can be increased linearly with the number of transmit antennas N_t (Foschini and Gans 1998). On the other hand, transmit/receive diversity schemes are impressively effective in increasing the diversity gain where consequently performance is improved (Foschini, 1996). In this paper, we restrict our work on spatial multiplexing systems and their related detection schemes due to the strong need for such systems in the 4G technology. Spatial multiplexing detection schemes can be mainly classified to linear, nonlinear and tree search. In this paper, performance comparison between these schemes is introduced.

II. System model

In this study, a conventional MIMO SM system with N_t transmit antennas and $N_r \geq N_t$ receive antennas has been considered. This model is a part of spatial multiplexing system such as VBLAST (Wolniansky et al., 1998), where the i^{th} data steam x_{Nt} is directly transmitted on the i^{th} transmit antenna. Then the received vector is given by

$$r = Hx + n \qquad (1)$$

with the $N_t \times 1$ transmit vector $x \triangleq (x_1, x_2, \ldots, x_{Nt})^T$, the $N_r \times N_t$ channel matrix H, the $N_r \times 1$ received vector $r \triangleq (r_1, r_2, \ldots, r_{Nr})^T$, and the $N_r \times 1$ noise vector $n \triangleq (n_1, n_2, \ldots, n_{Nr})^T$. The data streams x_{Nt} are assumed zero-mean with variance σ^2. The channel matrix H is considered perfectly known at the receiver. The noise elements are drawn from independent and identically distributed (i.i.d.) circular symmetric Gaussian random variables.

III. Spatial Multiplexing and Detection Problem

Spatial Multiplexing (SM) seems to be the ultimate solution to increase the system capacity without the need to additional spectral resources (Telatar, 1999; Bolcskei et al., 2006). The main challenge in MIMO SM system is the design of detection code with acceptable complexity and achieved performance (Bessai, 2005). A variety of detection techniques (Bolcskei et al., 2006) including linear, successive, tree search can be used to remove the effect of the channel and recover the transmitted data, see (Xu and Murch, 2002; Golden et al., 1999; Gore et al., 2002; Miyazaki et al., 2011). Maximum

Likelihood Detector (MLD) is considered as the optimum detector for the system of (1) that could effectively recover the transmitted signal based on the following minimum distance criterion. Although MLD achieves the best performance and diversity order, it requires a high complexity brute-force search. In order to solve the detection problem in MIMO systems, research has been focused on sub-optimal detection techniques that are efficient in terms of both performance and computational complexity. Two such techniques are Sphere Decoding (SD) and QR Decomposition with M-algorithm (QRD-M) which utilize restrict tree search mechanisms. These algorithms and more linear and non-linear detection techniques will be described and discussed next.

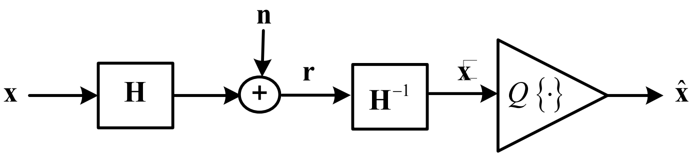

Figure 1: MIMO SM with linear receiver

IV. Linear Detection Techniques

The idea behind linear detection techniques is to linearly filter received signals using filter matrices, as depicted in Figure 1. An estimate of the transmit vector x is calculated as $\tilde{x} = Gr$, where G is the filtering matrix. The detected data is then obtained as $\hat{x} = Q\{\tilde{x}\}$, where $Q\{\cdot\}$ is the quantization function. The zero-forcing (ZF) detector is given by the pseudo-inverse of H , i.e., $G = H^{\dagger}$, then the result of ZF detection is

$$x_{ZF} = H^{\dagger} r = H^{\dagger}\left(Hx + n\right) = x + \tilde{n} \qquad (2)$$

which is the transmit vector x corrupted by the transformed noise $\tilde{n} = H^{\dagger} n$. This component makes ZF suboptimal due to the expected huge amplifications of the noise term. The minimum mean square error (MMSE) detector is given by (Kay, 1993)

$$G = \left(H^{H} H + \sigma^{2} I\right)^{-1} H^{H} \qquad (3)$$

which minimize the mean-square error $E\left\{\left\|\hat{x} - x\right\|^{2}\right\}$. Thus, the result of MMSE detection is

$$x_{MMSE} = \left(H^{H} H + \sigma^{2} I\right)^{-1} H^{H} r \qquad (4)$$

Figure 2 shows performance estimation of the linear detectors; the simulations are done for a $(N_t, N_r) = (4,4)$ system with QPSK modulation. The E_b / N_o, ranges between 0 dB and 30 dB. In this case MMSE curve performs better than ZF by about 5 dB at an error rate of 10-3. Both the ZF and MMSE detectors show a diversity order of more than $N_r - N_t + 1$, but less than N_r (Jankiraman, 2004). The linear detection schemes are favorable in terms of computational complexity, but their BER performance is severely degraded due to the noise enhancement in the ZF case, and when the channel matrix is ill-conditioned.

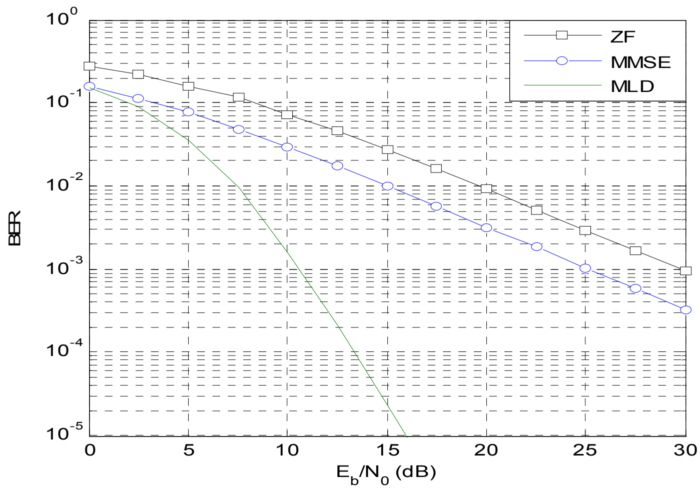

Figure 2: BER of linear detection algorithms

V. Successive Interference Cancellation

Although linear detection techniques are easy to implement, they lead to high degradation in the achieved diversity order. Another approach that takes advantage of the diversity potential of the additional receive antennas, Successive Interference Cancellation (SIC) (for instance, V-BLAST decoder). V-BLAST uses a serial decision-feedback approach to detect each layer separately (e.g. Wolniansky et al., 1998). The V-BLAST algorithm utilizes the already detected symbol x_i, obtained by the ZF or MMSE filtering matrix, to generate a modified received vector with x_i cancelled out. Thus the modified received vector becomes with fewer interferers and the performance improved due to a higher level of diversity. Error propagation can be a problem because incorrect detected data actually increase the interference when detecting subsequent layers. To reduce the effect of error propagation and to optimize the performance of VBLAST technique, it has been shown in (Golden et al., 1999) that the order of detection can increase the performance considerably. It is optimal to start detecting the components of x that suffer the least noise amplification. When the ordering is used the algorithm is called sorted ZF-VBLAST (SZF-VBLAST). The ZF-based solution in general is an easier solution but not optimum as it enhances the noise. Instead we have used the MMSE method, which gives us better performance. MMSE suppresses both the interference and noise components, whereas the ZF algorithm removes only the interference components. The algorithm is called sorted MMSE-VBLAST (SMMSE-VBLAST) when the ordering strategy is used. The main drawback of the VBLAST detection algorithms lies in the computational complexity, because multiple calculations of the pseudo-inverse of the channel matrix are required (Tse and Viswanath, 2005).

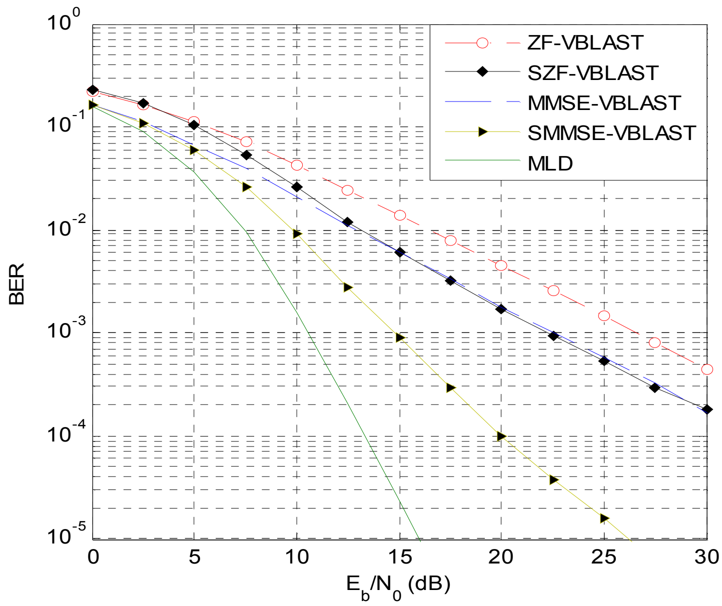

Figure 3 BER of VBLAST detection schemes

Figure 3 shows the performance of various VBLAST detection schemes that utilizing both ZF and MMSE criteria with and without using optimal ordering. At a target BER of 10^{-3} the difference between ZF-VBLAST curves is about 4 dB and the difference between MMSE-VBLAST curves is about 7 dB. This demonstrates the impact of employing signal ordering. Note that the performance advantage of the MMSE is quite considerable in all cases. The sorted MMSE-VBLAST lags the MLD curve by about 6.7 dB at a target BER of 10^{-4}.

VI. QR Decomposition Based Detection

The main computational bottleneck of the VBLAST algorithm is the multiple calculations of the pseudo-inverse of MIMO channel at each detection step. This can be avoided using QR Decomposition (QRD) based algorithm. In (Tse and Viswanath, 2005; Wubben et al., 2001), it was shown that QRD requires only a fraction of the computational efforts required by the V-BLAST. The QRD of the channel matrix H was introduced in (Gentle, 1998). It was shown that VBLAST algorithm can be restated in terms of QRD of the channel matrix $H = QR$ (Wubben et al., 2001; Wubben et al., 2002; Biglier et al., 2002; Bohnke et al., 2003), where H is decomposed into a $N_r \times N_t$ unitary matrix Q , i.e., $Q^H Q = I$ and a $N_t \times N_t$ upper triangular matrix R . Then, the received vector r in (1) is multiplied with Q^H (Shiu and Kahn, 1999),

$$r = H \cdot x + n = QR \cdot x + n$$

$$Q^H r = Q^H QR \cdot x + Q^H n$$

$$y = R \cdot x + v \qquad (5)$$

Given the upper triangular structure R , the k^{th} element of y is given by

$$y_k = R_{k,k} x_k + \sum_{i=k+1}^{n_T} R_{k,j} \hat{x}_i \qquad (6)$$

and is totally interference-free. Thus it can be used to estimate x_{Nt} ;

$$\hat{x}_{Nt} = Q\left[\frac{y_{Nt}}{r_{NtNt}}\right] \qquad (7)$$

Detecting $(k = N_t - 1, \ldots, 1)$ is carried out equivalently, noting that already-detected components of x are cancelled out from the received vector. These procedures are repeated up to the first component x_1.

As mentioned in section V, the detection sequence is critical due to the risk of error propagation. Following the same idea as VBLAST, the symbols can be detected in order of decreasing SNR. This requires rearranging the columns of H in increasing order of 2-norm so that the last symbol corresponding to the last column gets detected first and so on. The optimal ordering can be determined just by permuting the columns of x according to the elements of p (where p is the permutation vector). In MMSE-QRD case, the channel matrix H should be extended to

$$\tilde{H} = \begin{bmatrix} H \\ \sigma_n I \end{bmatrix} \quad (8)$$

and decomposed into Q and R matrices such that $\tilde{H} = QRP$, with P as the permutation matrix. Noting that when the ordering is used, the algorithm is labelled sorted QRD (SQRD) as depicted in Figure 4.

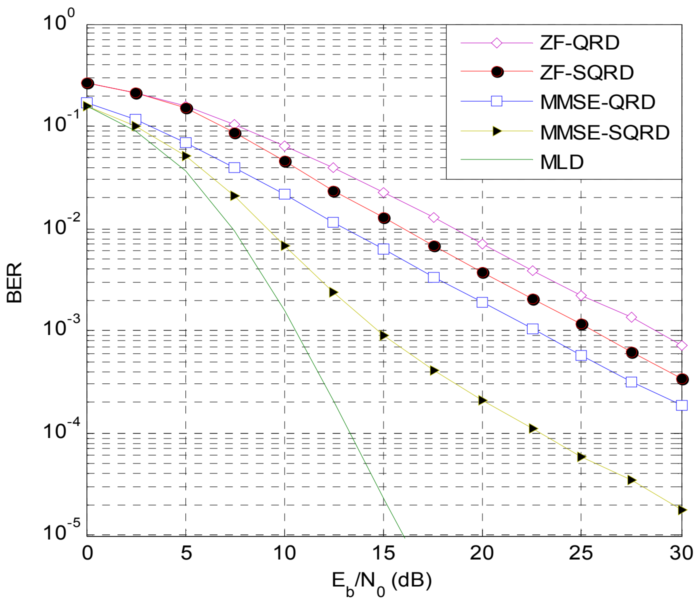

Figure 4 BER of QRD detection schemes

Figure 4 shows the BER performance of the QRD-based schemes. It can be seen that the MMSE-based perform better than ZF in all cases. At target BER of 10^{-3}, MMSE-QRD leads both ZF-QRD and ZF-SQRD by about 5.5 dB and 2.5 dB respectively. The best performance is achieved by MMSE-SQRD scheme, where it lags the optimum performance by about 9 dB at target BER of 10^{-4}.

VII. Tree-Search Detection Techniques

Several tree-search detection algorithms have been proposed in the literature that achieve quasi-ML performance while requiring lower computational complexity. In these techniques, the search problem of MLD is presented as a tree where nodes represent the symbols' candidates. In the following, we introduce two tree-search algorithms and discuss their advantages and drawbacks.

A. Sphere Decoding

Sphere Decoding (SD) approach was inspired from the mathematical problem of computing the shortest nonzero vector in a lattice (Viterbo and Biglieri, 1993). SD algorithm was originally described in (Pohst, 1981) and refined in (Fincke and Pohst, 1985) to substantially reduce the computational complexity of signal detection in MIMO systems. In SD, the search can be restricted to be in a circle with a radius d around the received signal r (Damen et al., 2000; Agrell et al., 2002), therefore

$$\hat{\mathbf{x}}_{SD} = \arg\min_{\mathbf{x}\in\Omega^{n_T}} \left(\left\| \mathbf{y} - \mathbf{R}\mathbf{x} \right\|^2 \le d^2 \right) \quad (9)$$

According to the analysis in (Hassibi and Vikalo, 2001), SD can transform the ML detection problem into a tree search and pruning process and achieve quasi-ML performance. The SD can be considered as a depth-first search approach with tree pruning process (Vikalo et al., 2006). In SD algorithm, the most important issue is the strategy based on which hypotheses are tested per level. For the detection problem of (9), the hypotheses should meet the condition (Dai et al., 2005)

$$\sum_{j=1}^{N_t} \left| R_{j,j}\left(\hat{x}_j - x_j\right) + \sum_{i=j+1}^{N_t} R_{j,i}\left(\hat{x}_i - x_i\right) \right|^2 \le d^2 \quad (10)$$

And the accumulative metric in (10) is then calculated successively, where the metric at the N_t detection level is given by:

$$E_{N_t} = \left(y_{N_t} - R_{N_t,N_t}\hat{x}_{N_t} \right)^2 \quad (11)$$

Worth to note that the hypotheses can be tested based on two strategies; the Fincke-Pohst and the Schnorr-Euchner (Qingwei, 2008). For ease of understanding below is a numerical example. The search tree generated by the SD algorithm is given in Figure 5 for the case when $m = 3$, $\Omega = \{+1, -1\}$, and $d^2 = 3$, where y and R in (9) are given by

$$y = \begin{bmatrix} 0 \\ 3.8 \\ -1.1 \end{bmatrix}, \text{ and } R = \begin{bmatrix} 0.4 & -1.2 & -2.7 \\ 0 & 0.5 & -2.7 \\ 0 & 0 & 0.6 \end{bmatrix}$$

Each candidate symbol, $x \in \Omega$, is indicated by a leaf node in the tree. The metric of each node, given by the left hand side of (10), is indicated by the number to the right of each node. Each node with a metric less than d^2 is included in the search and indicated in black. On the other hand the white nodes are not visited by the SD algorithm. The ML estimate, $x_{ML} = [-1 \quad +1 \quad -1]$, has an objective value of 1.82 in (9) which is also the smallest node value.

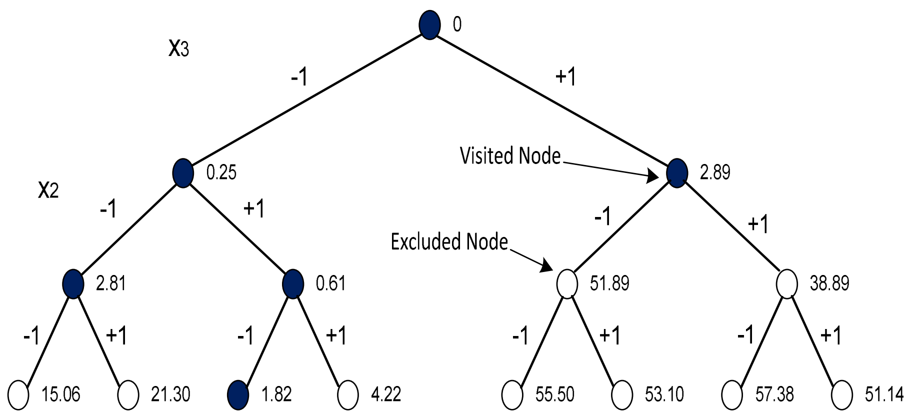

Figure 5: Example illustrating SD search tree. Nodes visited by the algorithm are shown in black

As indicated by Figure 5, the total number of nodes visited is usually much smaller than the set of all symbol vectors Ω^m, which implies that the SD algorithm is of substantially lower complexity than the brute force search.

B. QRD-M Detection

QRD-M was proposed to achieve quasi-ML performance while requiring fixed computational effort. QRD-M algorithm was originally discussed in (Kim and Iltis, 2002) and was first used in signal detection in MIMO system in (Yue et al., 2003). QRD-M algorithm can reduce the tree search complexity by selecting only *M* candidates at each layer instead of testing all the hypotheses of the transmitted symbol (Kim et al., 2005). These *M* candidates are the smallest accumulated metric values. QRD-M Algorithm can be considered as a breadth-first search that has only one searching strategy. Basically, the idea of QRD-M Algorithm is similar to SQRD approaches for MIMO detection (section VI). However, instead of selecting only the closet constellation point in each layer, a total of *M* metrics are considered in evaluation. The algorithm starts by applying the QR decomposition to (1) then the ML detection problem can be reformulated as

$$\hat{x}_{ML} = \arg\min_{x \in \Omega^{N_t}} \left\| y - Rx \right\|^2 , \qquad (12)$$

and for the tree depth of i , $1 \le i \le N_t$ the metric for each branch is

$$\left| \tilde{y}_{Nt-i+1} - R_{Nt-i+1} \hat{x}_i \right|^2 \qquad (13)$$

where $\tilde{y}_i$ is the i^{th} element of $\tilde{y}$, and R_i is the i^{th} row of R , and $\hat{x}_i$ is the vector of the correspondence nodes of the particular branch. For the ease of understanding, the QRD-M algorithm can be summarized in the following six main steps:

1. Perform QRD on *H*
2. Pre-multiply *y* with Q^H
3. Extend all branches to $M\,\Omega$ nodes
4. Calculate the branch metrics using (14)
5. Order the branches according to their metrics, retaining only *M* branches and discarding the rest
6. Move to next layer and go to step 3

Figure 6 shows the BER performance of SD and QRD-M algorithms in 4 x 4 MIMO SM system. SD algorithm overlap MLD performance and the QRD-M algorithm achieves the ML performance for M = |Ω| which equals 4 in the case of 4-QAM. It is remarkable that the tree-search

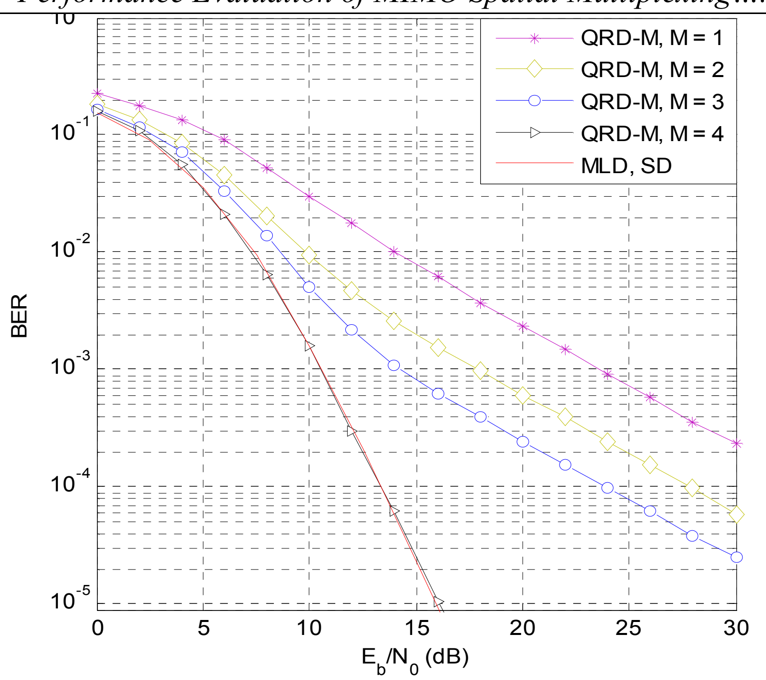

Figure 6 BER of SD and QRD-M performance for several values of M

based detection techniques provide quasi-ML performance and the QRD-M algorithm in particular is the most amenable to hardware implementation. It should be noted that the detection complexity of SD and QRD-M was significantly higher than that of linear and SIC detection algorithms.

VIII. Conclusion

In this study a variety of the MIMO SM detection schemes have been described, discussed and compared in terms of performance and computational complexity.

Different performance simulations have been generated for each detection categories to investigate and evaluate their BER. It has been shown that the linear detection techniques have poor performance due to the huge amplification in noise power in ZF-case. The ordering strategy involved VBLAST has important benefits but the performance improvement is limited due to error propagation. This error propagation has been alleviated by QRD algorithms. The tree-search based detection techniques; i.e., SD and QRD-M with the two promising approaches. SD has achieved MLD performance. In case of QRD-M, while the number of survival candidates increases the performance converges to that of MLD.

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