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# Krapivsky-Redner modification of nonlinear Barabási-Albert networks

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**Abstract:**

*In growing Barabási-Albert (BA) networks, a new node randomly selects an existing target node and attaches to it randomly with a probability r proportional to the number k of neighbors already attached to the target node. Krapivsky and Redner use, also for different networks: "a new node randomly selects an existing target node, but attaches to a random neighbor of this target." In nonlinear BA networks, r is made proportional to $k^{\alpha}$ with $\alpha = 1$ for the standard BA case. We simulate here nonlinear Barabási-Albert-Krapivsky-Redner (BAKR) networks, where r is applied to the selection of the target, not to the selection of the target neighbor. We use undirected Barabási-Albert networks. For the maximum number kmax of neighbors we find little effect from α, while the distribution n(k) of the number of neighbors has a normal power law and there is no gap or strong peak in the number of neighbors k(i). All this contradicts our earlier simulations without redirection.*

**Keywords:** Barabasi Albert network, probability, number of neighbors, nodes.

## Introduction:

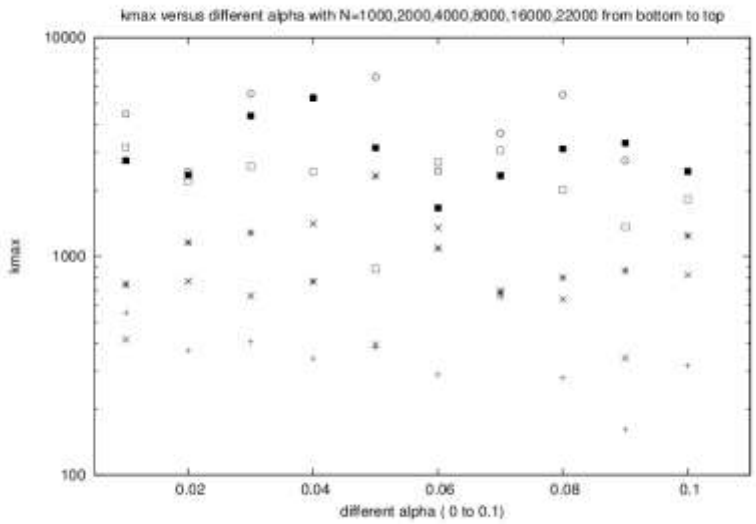
Barabási Albert networks (BA) and their modifications have been studied for several years[1,2], We studied the nonlinear Barabási-Albert network (NLBA) where a new node connects to a vertex having k neighbors with a probability r [3] proportional to $k^{\alpha}$ real; usually $\alpha = 1$. Each new node adds m new edges to the network, and we developed two versions from this model NLBA1 and NLBA2 [4,5]. We now study the modification of Krapivsky and Redner who use, also for different networks: "a new node randomly selects an existing target node, but attaches to a random neighbor of this target" [6,7].

Krapivsky-Redner networks use redirection as a fundamental network growth mechanism to determine how a new node n attaches to a growing network. For undirected networks, without a prescribed direction for each link, redirection is implemented as a new node n chooses from all already existing nodes a provisional target node at random, with probability $0 < r < 1$. Then a randomly selected neighbor of the target attaches to the ancestor n of the target. We now investigate nonlinear Barabási-Albert-Krapivsky-Redner (BAKR) networks, where r is applied to the selection of the target, not to the selection of the target neighbor. We use undirected nonlinear Barabási-Albert networks.

## Data and Simulation:

In our simulations we use the Fortran program of BAKR as in the appendix. In the simple version, it uses a rectangular array neighb(maxnb,max), where max = N + m is the network size and maxnb the maximum number of neighbors for a single node.
With small networks, we start with a parameter maxnb = 1.1*maxtime*m, but in large networks, we use maxnb = 0.9*maxtime*m, since we must adjust the parameter maxnb with small N such that it is never smaller than the maximum kmax of the number of neighbors k(i) over all node indices i. With maxnb=max*m one has the absolute maximum: each of the max new nodes n produces m new neighbor bonds. However, our first tests show that kmax is much smaller, and then one can save memory by using a smaller parameter maxnb. When we vary $\alpha$ between 0.1 and 1.9, with

m = 4, and observe values of kmax, of the number of neighbors k(i) of node i, and of the number n(k) of nodes with k neighbors, we notice no gaps, no strong peaks, and normal power law in the n(k) , and kmax varies little with varying values of α. We simulate different values of lattice size (N= 1000,2000,4000,8000,16000,22000) with α up to 0.1 with constant values of m=4. For N=16000 and 22000 nodes, we used Allocatable command of fortran90 because maxnb is large. We use a logarithmic scale at the y-axis (kmax) to let results become clearer, and show kmax for small and large N in figure (1).




Figure(1): kmax versus α up to 0.1 for different size N

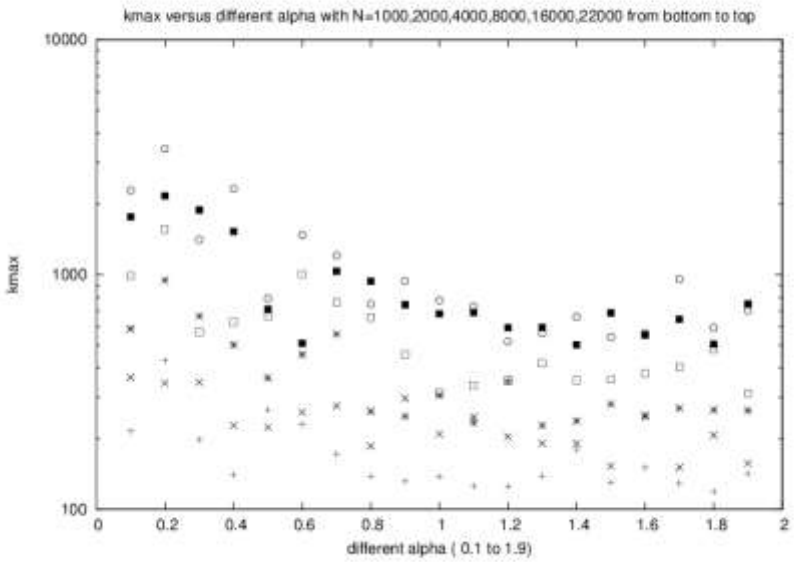
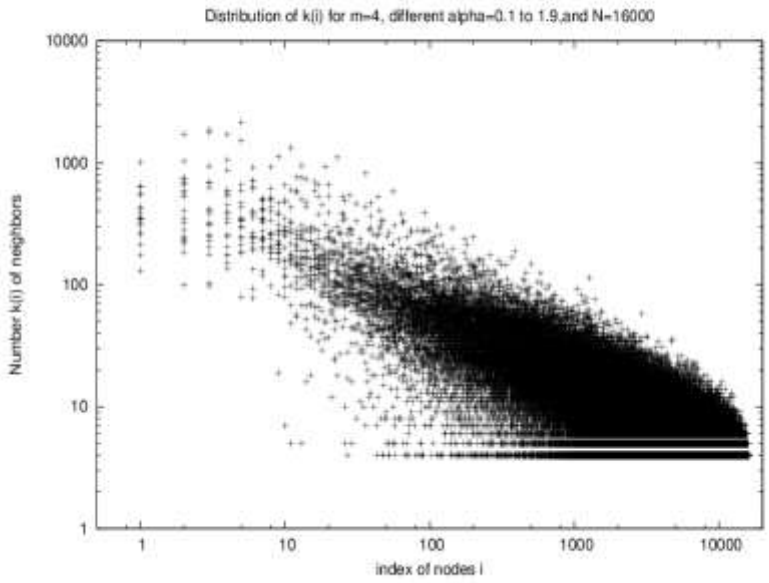


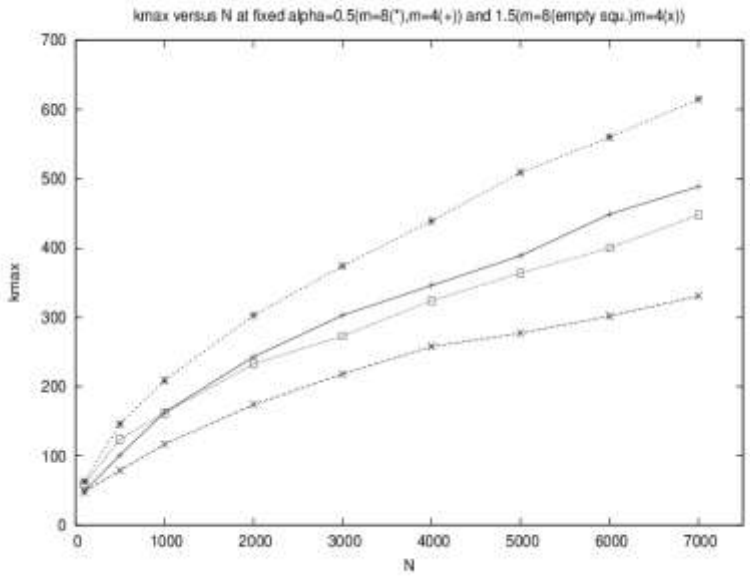

Figure (2) shows kmax versus α for larger values of α up to 1.9.

Now we fix N=22000, m=4, but change α from 0.1 to 1.9, and we get figure( 3) which shows the number of k(i) of neighbors versus index of nodes i.




Figure(3): number k(i) of neighbors versus index of nodes I with m=4 and N=16000 as example.

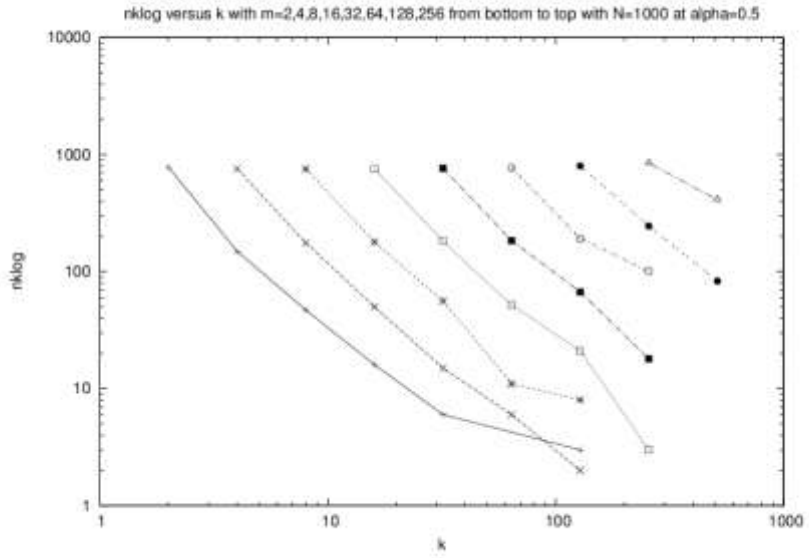
When we fix α at 0.5 and 1.5 with changing size N from 1000 to 7000 at constant m=2,4,8,16,32,64,128,256. We get figure 4 . It shows that kmax increases roughly proportional size N at fixed α and number m of neighbors.




Figure(4): kmax versus size N versus at **α** 0.5 [m=8(*) m=4(+)] and **α** 1.5[m=4(empty squ.) m=8(x)].

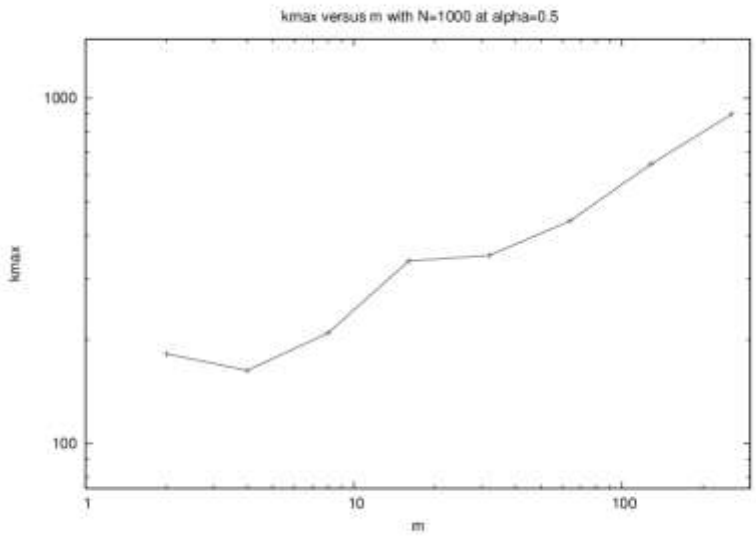
We see that kmax is always much smaller than N, in particular there is no strong dependence of kmax on **α,** while in [4] we found kmax to increase strongly up to close to the theoretical maximum N*m for α > 1.5.

We simulate many values of m=2,4,8,16,32,64,128,256 as we can in our computers, for small **α**=0.5 and with size = 1000, and we take nklog is the sum over all n(k) within an interval proportional to k, the n(k) vary roughly as 1/k**3 , while the slope in our curves in figure(5) is close to -2.




Figure(5): n(k) versus k versus at $\alpha$ 0.5 [m2,4,8,16,32,64,128,256 from bottom to top with N=1000)].

Finally we plot figure 6 as kmax versus different m which shows us the relation between kmax and m which is: kmax proportional directly to m.




Figure(6): kmax versus m at **α** 0.5 with multiple of m

## Conclusion:

We observed no gaps in k(i), no peaks in k(i), no strong variation of kmax versus **α** with the Krapivsky-Redner modification. We see that kmax is always much smaller than N, in particular far below the theoretical maximum N*m. All these results differ from our earlier simulations without redirection [3,4,5].

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