Reliability Function of the Consecutive 2-within-(2,2)-out-of-(m,n) : F System

دالة موثوقية (احتمال عمل) النظام التتابعي ثنائي البعد 2-within-consecutive-(2,2)-out-of-(*m*,*n*): F

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Abstract:

The two-dimensional consecutive systems have been used for many engineering systems, the thin film transistor liquid, connector pins for an electronic device, the diagnostic disease X-ray system, the super vision system, the reactor, etc., one of the most important system is the 2-within-consecutive-(2,2)-out-of-(m,n): F linear (circular) system.

The 2-within-consecutive-(2,2)-out-of-(m,n): F linear (circular) system consists of $m \times n$ *components, it fails if there are at least 2 failed components within any square of* 2×2 *components. This paper computes the reliability function of the system using a recursive algorithm, which depends on the working states of the consecutive-2-out-of-n: F linear and circular system. The proposed algorithm requires* $O(m2^{2n-2})$ *computing time, which is little less than that in the literature. Illustrative example of the system was provided with independent identical distributed components, but also it is applied for the independent non-identical case.*

Keywords: One dimensional consecutive system, Two-dimensional consecutive system, Reliability.

الملخص:

يتكون النظام التتابعي الشبكي ثنائي البعد المسمى (2−within−consecutive−(2,2)−out−of−(m,n): F)، من عدد $m \times n$ من المكونات ، يفشل هذا النظام عندما يفشل على الاقل مكونين من اصل 2×2 مكون، و النظام التتابعي الشبكي ثنائي البعد نوعان دائري وخطي.

تقدم هذه الورقة خورازمية تكرارية (طريقة استرجاع الرتبة الاقل) لحساب دالة موثوقية (احتمال عمل) هذا النظام، عن طريق استخدام حالات الفشل والعمل للنظام احادي البعد التتابعي (2−out−of−n: F) الخطي والدائري، الخوارزمية تحتاج الى $O(m2^{2n-2})$ عدد مرات حساب، والتي هي اقل بقليل من الخورازميات المشابهة في ادبيات البحث في هذا المجال، لقد تم تقديم مثال توضيحي للخورازمية عندما تكون مكونات النظام ذات توزيعات احتمالية مستقلة ومتطابقة، ولكن الخوارزمية تصلح عندما تكون المكونات ذات توزيعات احتمالية مستقلة وغير متطابقة.

1. Introduction

The consecutive *k*-out-of-*n*: F system consists of *n* connected components is a one-dimensional consecutive system, it fails if at least *k* consecutive components fail, it is classified according the connection between components into two types: linear and circular. (Chang and Niu 1981) introduced the system, but it had mentioned previously by (Kontoleon 1980) under the name "*r*-successive-out-of-*n*: F system". many engineering systems have been used the model of consecutive *k*-out-of-*n*: F linear and circular system (e.g. oil pipeline system, telecommunication systems, spacecraft relay stations, vacuum system of the electron accelerator, the photographing of a nuclear accelerator, microwave stations and quality control problems, etc.). Later, abundant researches studied its generalizations and reliability function, optimal design, reliability bound and importance, which achieved in (Derman *et al*. 1982, Sfakianakis and Papastavridis 1993, Malinowski J., Preuss 1995, Chao *et al*. 1995, Chang *et al*. 1998, Yamamoto H., and Akiba 2003, Nashwan 2015, Cai et al. 2016, Gökdere *et al*. 2016, Cai *et al*. 2019, and Zhang *et al*. 2019).

One of these generalizations, is "the consecutive-*k*-within-*m*-out-of-*n*: F system" which was introduced by (Griffth 1986) despite it had been mentioned by (Tong 1985). The system fails if there are *m* consecutive components, which are including among them at least k failed components. There are many system models can be considered as a special case of it, the consecutive *k*-out-of-*n*: F system when *k*=*m*, the simple *k*-out-of-*n*: F system, when *m*=*n*, the series system when *k*=1, and the parallel system when *k*=*n*. (Sfakianikis *et al*. 1990) and Papastavridis and (Koutras 1993) were proposed efficient lower and upper bounds. Moreover, (Iyer 1992) studies the life time of the system, as well as many researchers were studied its reliability (Derman *et al*. 1982, Bohme *et al*. 1992, Sfakianakis *et al*. 1992, Malinowski and Preuss 1995, Habib and Szantai 2000, Eryilmaz 2012, Endharta *et al*. 2016, Nashwan 2018, Nashwan 2020). The system is applied to quality control and inspection procedures, non-random clustering, radar detection and sliding-window detection systems.

After that, the two-dimensional version of consecutive system was introduced by (Salvia & Lasher 1990), under the name "consecutive k^2-out-of-n^2: F system", the system consists of n^2components, the components are distributed and connected like a $n \times n$ square grid, each side has *n* components; the system fails if there is at least $k \times k$ sub square grid that contains all failed components. (Bohme *et al*. 1982), generalized the notion of the 2-dimensional consecutive systems, to the connected-***X***-out-of-(*m*,*n*): F system, ***X*** is the shape of a sub region of failures that lead the system to fails. In the linear type, the system consists of $(m \times n)$ components (connected and distributed on a lattice as $m \times n$ matrix, the matrix elements represent the components, the *m* rows includes *n* components, and the *n* columns have *m* components, while the circular system consists of *m* circles and *n* rays, the circles have the same center, each circle includes *n* components, and each ray have *m* components, the components located at the intersection points between the path of the rays and the circles, see (Yamamoto and Akiba 2003, Nashwan 2015, Cowell 2015, and Nashwan 2019).

(Papastavridis and Koutras 1993) proposed the 2-dimensional *k*-within-consecutive-(*r*,*s*)-out-of-(*m*,*n*) linear and circular system, it fails when $\{\boldsymbol{X}= k\text{-within-}(r,s)\}$, i.e. any k components fail among any $r \times s$ submatrix (a grid of $r \times s$ components in the circular system). The system has many important applications in our daily life (the Thin Film Transistor Liquid Crystal Display failure model. The XGA (1024×768 = total 786432 dot) TFT display system fails if and only if more than or equal to 10 dots fail in 10×10 dot matrix, then the system rectangle 10-within (10,10)-out-of-(1024, 768): F system). (Akiba and Yamamoto 2001) as well as (Lin and Zuo 2000) presented recursive algorithms for exact reliability for such system, while (Chang and Huang 2010) computed the reliability of the system using a finite Markov chain.

It is worth mentioning that, the computation of 2-dimensional consecutive system reliability is very complicated, the familiar with the literature knows that all the previous methods were very complicated, therefor, many researchers were studied the reliability boundaries, optimal arrangement, and a special cases of the k-within-consecutive-(r,s)-out-of-(m,n) linear and circular system. (Nashwan 2016) studied a special case of the system when $k = r = s = 2$ and computed the reliability using a Markov chain, (El Sayed 2009) studied the failure probability function of the system when n=2,3,4. while (Makri and Psikallis 1997) provided an upper and lower bound of the its reliability.

This paper introduced a recursive algorithm to compute the reliability function of the 2-within-consecutive-(2,2)-out-of-(m,n): F linear (circular) system [denoted L(C)(2,m,n) system], where it is structured as follows: In the second Section, the consecutive 2-out-of-n: F system is reviewed, and studied its failure and working states, the third section evaluated the reliability the L(C)(2,m,n) system using the working and the failure states in the consecutive 2-out-of-n: F system. Throughout, the paper assumes that (the system has a mutually statistically independent components, and they are either “failed” or “working” states, as well as the system), Table 1 explains the used acronyms and notations.

Table 1: Acronyms and Notation

$\mathbb{I}_j^i$	: $=\{i, i+1, \ldots, j\}$
$[y]$	: The greatest integer number of y.
$P(\mathbb{I}_n^1)$	: The power set of $\mathbb{I}_n^1$.
f^α	: The composite function α times, where $f(x) = x \, mod \, n + 1 : x \in \mathbb{I}_{n-1}^1$
$p_X(q_{WX})$	: Reliability of the set W where $p_{WX} = \prod_{j \notin X} p_i \, , q_X = \prod_{j \in X} q_j$
$R(X)$	: $= P\{Z_i = X\} = p_X q_X$
$R_{L(C)}(i, X)$	: The reliability of the subsystem L(C)(2,*i*,*n*) when X represents the failed components in the i^{th} layer (circle).
$R_{L(C)}(i)$	: The reliability function of the subsystem L(C)(2,*i*,*n*)

2. The Consecutive 2-out-of-*n*: F Systems

Consider the consecutive 2-out-of-*n*: F system, it consists of n components and fails when any 2 consecutive components fail. If $\mathbb{I}_n^1$ denotes indices of the system components, and the system is represented by the set $X = \{x_1, x_2, \ldots., x_j\} \subseteq \mathbb{I}_n^1$, the element of X refers the indices of the failed components, such that $x_i < x_h$ for all $1 \le i < h < j \le n$. X is a failure state, if X includes any two consecutive indices, for example, assume $X = \{1,3,5,7\}$ represents the consecutive-2-out-of-7-out-of-*n*: F system, the system consists of 7 components, the odd components are only the failed components, while the even components are in the working state. Moreover, X is a working state when the system is linear, and a failure state if the system is circular.

Define $\Phi^{\mathrm{L(C)}}(\Gamma^{\mathrm{L(C)}})$ is the collection of all working (failure) states of the consecutive 2-out-of-*n*: F linear (circular), then $\mathrm{P}(\mathbb{I}_\mathrm{n}^1) = \Phi^{\mathrm{L(C)}} \cup \Gamma^{\mathrm{L(C)}}$, where $\Phi^{\mathrm{L(C)}} = \{X \in P(\mathbb{I}_n^1) : \mathbb{I}_{r+1}^r \not\subset X\}$ such that $r \in \mathbb{I}_{n-1}^1$in the linear type, and for the circular one $r \in \mathbb{I}_n^1$ where $n + 1 \equiv 1$. Note that Ψ^{L} is a sub collection of Ψ^C, due the connection between the 1st and the n^{th} components vice versa $\Phi^{\mathrm{C}} \subseteq \Phi^{\mathrm{L}}$.

3. Reliability Function of the L(C)(2,*m*,*n*) System.

Consider the L(C)(2,*m*,*n*) system, the system fails, if any 2 components among any sub matrix 2×2 fails. In fact, the system fails if one of the following events occurs (see figure 1):

1. Any layer (or circle) in the system has 2 consecutive failed components,
2. The indices of the failed components in any layer (or circle) are far away from those in the next layer (or circle) less than 2 steps.

For any layer (circle) in the system, denotes the components by $\mathbb{I}_n^1$, and the failed components of any two consecutive layers (circles) are represented by $X, Y \in P(\mathbb{I}_n^1)$. To avoid the first event of failure above, X should be a working state in the consecutive 2-out-of-*n*: F linear (circular) system i.e. $X \in \Phi^{\mathrm{L(C)}}$, otherwise, there are two consecutive failed components on the same layer (circle), which implies that the whole system is going to the failure state. To avoid the second event of failure, $|x-y| \geq 2, \forall x \in X, y \in Y$, in the linear system, and $Y \cap \left[\bigcup_{\alpha \in \{1,n-1,n\}} f^{\alpha}(X)\right] = \emptyset$ in the circular type, see the following figures.

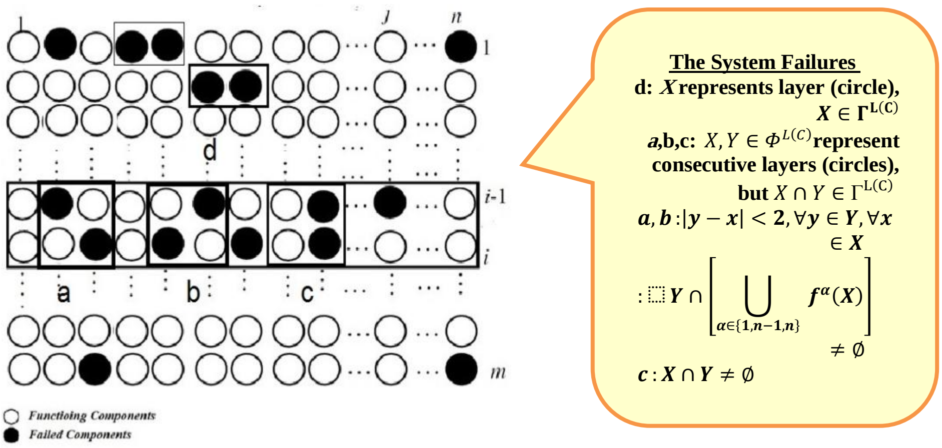

Figure 1. The failure states of the L(C)(2,*m*,*n*) system.

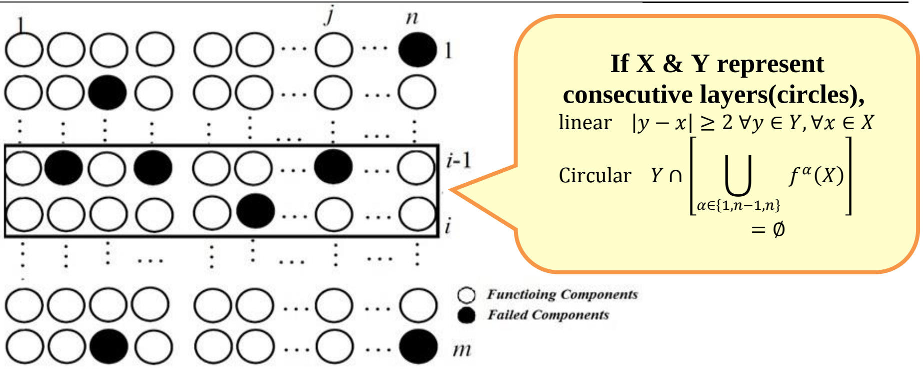

Figure 2. A working state of the L(C)(2,*m*,*n*) system.

In this context, let Z_i be a random variable fixes the state of the i^{th} layer (circle) such that, the event that $\{Z_i = X\}$ is the event that "the i^{th} layer (circle) has the failed components denoted by $X \in P(\mathbb{I}_n^1)$", and $A_i^{L(C)}(X)$ denotes the event that "the subsystem L(C)(2,*i*,*n*) is in the working state, where the set $X \in P(\mathbb{I}_n^1)$ represents the i^{th} layer (circle)", which means that the $(i\text{-}1)^{th}$ layer (circle) may be any set $Y \in \Phi^{\mathrm{L(C)}}$ such that, X and Y have no common failed components or they have not any failed component before or after the failed components represented by X, (see figure 3.1, a, b and c).

Now, define $\Phi^{\mathrm{L}}(X) = \{Y \in \Phi^{\mathrm{L}} : |y - x| \geq 2\ \forall y \in Y\ \forall x \in X\}$ for the linear system, and for the circular system $\Phi^{\mathrm{C}}(X) = \left\{Y \in \Phi^{\mathrm{C}}, Y \cap \left[\bigcup_{\alpha\in\{1,n-1,n\}} f^{\alpha}(X)\right] = \emptyset\right\}$. Note that the elements of $\Phi^{\mathrm{L(C)}}(X)$ are consists of all elements that participate in the event $A_i^{L(C)}(X)$, i.e.

$$A_i^{L(C)}(X) = \begin{cases} \{Z_i = X\} & : i = 1 \\ \{Z_i = X\} \cap \left[\bigcup_{Y\in\Phi^{\mathrm{L(C)}}(\mathrm{X})} A_{i-1}(Y)\right] & : i = 2,3,\dots,m \end{cases}$$

Theorem 3.1: Consider the L(C)(2,*i*,*n*) subsystem, for *i*=1,2,…,*m* and , $X \in P(\mathbb{I}_n^1)$ then

$$R_{L(C)}(i,X) = \begin{cases} R(X) & i = 1, \text{ and } X \in \Phi_{L(C)} \\ R(X) \displaystyle\sum_{Y \in \Phi^{L(C)}(X)} R_{L(C)}(i-1,Y) & i \geq 2, \text{ and } X \in \Phi_{L(C)} \\ 0 & X \in \Gamma_{L(C)} \end{cases}$$

Where the reliability of the system is

$$R_{L(C)}(m) = \sum_{X \in \Phi^{L(C)}} R_{L(C)}(m,X)$$

Proof:

For *i*=1

$$R_{L(C)}(1,X) = P\{A_1(X)\} = P\{Z_1 = X\} = R(X) = p_X q_X$$

For *i*=2, 3,…,*m*

$$R_{L(C)}(i,X) = P\{A_i(X)\} = P\left\{\{Z_i = X\} \cap \left\{\bigcup_{Y \in \Phi^{L(C)}(X)} A_{i-1}(Y)\right\}\right\}$$

$$= P\left\{\bigcup_{Y \in \Phi^{L(C)}(X)} \{Z_i = X\} \cap \{A_{i-1}(Y)\}\right\}$$

$$= \sum_{Y \in \Phi^{L(C)}(X)} P\big\{\{Z_i = X\} \cap \{A_{i-1}(Y)\}\big\}$$

$$= P\{Z_i = X\}\left(\sum_{Y \in \Phi^{L(C)}(X)} P\{A_{i-1}(Y)\}\right)$$

$$= R(X) \sum_{Y \in \Phi^{L(C)}(X)} R_{L(C)}(i-1,Y)$$

For *i*=*m*

$$R_{L(C)}(m) = P\left\{\bigcup_{X \in \Phi^{L(C)}} A_m(X)\right\} = \sum_{X \in \Phi^{L(C)}} P\{A_m(X)\}$$

$$= \sum_{X \in \Phi^{L(C)}} R_{L(C)}(m,X)$$

Example 3.1: Compute the reliability of the 2-within-consecutive-(2,2)-out-of-(3,4): F circular system when the components are independent and identically distributed.

For convenience' sake write for example $X = 124$ instead of $X = \{1,2,4\}$,

$$P(\mathbb{I}_4^1) = \{\emptyset, 1,2,3,4,12,13,14,23,24,34,123,124,134,234,1234\}$$

$$\Phi^C = \{\emptyset, 1,2,3,4,13,24\}, \qquad \Phi^C(\emptyset) = \Phi,$$

$$\Phi^C(1) = \{\emptyset, 3\}, \quad \Phi^C(2) = \{\emptyset, 4\}, \quad \Phi^C(3) = \{\emptyset, 1\}, \quad \Phi^C(4) = \{\emptyset, 2\},$$

$$\Phi^C(13) = \{\emptyset\}, \qquad \Phi^C(24) = \{\emptyset\}$$

$$R_C(1,\emptyset) = R(\emptyset) = p^4$$

$$R_C(1,\{1\}) = R_C(1,\{2\}) = R_C(1,\{3\}) = R_C(1,\{4\}) = R(1) = p^3 q,$$

$$R(1,\{13\}) = R(\{13\}) = R(1,\{24\}) = R(\{24\}) = p^2 q^2$$

$$R_C(1) = \sum_{X \in \Theta^C} R_C(1,X)$$

$$= R_C(1,\emptyset) + R_C(1,\{1\}) + R_C(1,\{2\}) + R_C(1,\{3\}) + R_C(1,\{4\}) + R_C(1,\{13\}) + R_C(1,\{24\})$$

$$= p^4 + 4p^3 q + 2p^2 q^2$$

$$R_C(2) = \sum_{X \in \Phi^C} R_C(2,X)$$

$$R_C(2,\emptyset) = R(\emptyset) \sum_{Z \in \Phi^C(\emptyset)} R_C(1,Z) =$$

$$= R(\emptyset) \begin{bmatrix} R_C(1,\emptyset) + R_C(1,\emptyset) + R_C(1,\{1\}) + R_C(1,\{2\}) \\ + R_C(1,\{3\}) + R_C(1,\{4\}) + R_C(1,\{13\}) + R_C(1,\{24\}) \end{bmatrix}$$

$$= p^4[p^4 + 4p^3 q + 2p^2 q^2] = p^8 + 4p^7 q + 2p^6 q^2$$

$$R_C(2,\{1\}) = R(\{1\}) \sum_{Z \in \Phi^C(1)} R_C(1,Z)$$

$$= R(\{1\})[R_C(1,\emptyset) + R_C(1,\{3\})] = p^3 q[p^4 + p^3 q]$$

$$= p^7 q + p^6 q^2$$

$$R_C(2,\{2\}) = R(\{2\}) \sum_{Z\in\Phi^C(2)} R_C(1,Z)$$
$$= R(\{2\})[R_C(1,\emptyset) + R_C(1,\{4\})] = p^3q[p^4 + p^3q]$$
$$= p^7q + p^6q^2$$

Anyone can easily check that $R_C(2,\{1\}) = R_C(2,\{2\}) = R_C(2,\{3\}) = R_C(2,\{4\})$ and $R_C(2,\{13\}) = R_C(2,\{24\})$ where,
$R_C(2,\{13\}) = R(\{13\}) \sum_{Z\in\Theta^C(13)} R_C(1,Z) = R(\{13\})[R_C(1,\emptyset)] = p^2q^2[p^4] = p^6q^2$,
Then

$$R_C(2) = \sum_{X\in\Phi^C} R_C(2,X)$$
$$= R_C(2,\emptyset) + R_C(2,\{1\}) + R_C(2,\{2\}) + R_C(2,\{3\}) + R_C(2,\{4\}) + R_C(2,\{13\}) + R_C(2,\{24\})$$
$$= [p^8 + 4p^7q + 2p^6q^2] + 4[p^7q + p^6q^2] + 2[p^6q^2]$$
$$= p^8 + 8p^7q + 8p^6q^2$$

$$R_C(3) = \sum_{X\in\Phi^C} R_C(3,X)$$
$$R_C(3,\emptyset) = R(\emptyset) \sum_{Z\in\Phi^C(\emptyset)} R_C(2,Z)$$

$$= R(\emptyset)\begin{bmatrix} R_C(2,\emptyset) + R_C(2,\{1\}) + R_C(2,\{2\}) + R_C(2,\{3\}) + R_C(2,\{4\}) \\ +R_C(2,\{13\}) + R_C(2,\{24\}) \end{bmatrix}$$
$$= p^4[p^8 + 8p^7q + 8p^6q^2] = p^{12} + 8p^{11}q + 8p^{10}q^2$$

Anyone can easily check that $R_C(3,\{1\}) = R_C(3,\{2\}) = R_C(3,\{3\}) = R_C(3,\{4\})$ and $R_C(3,\{13\}) = R_C(3,\{24\})$ where,

$$R_C(3,\{1\}) = R_C(3,\{2\}) = R_C(3,\{3\}) = R_C(3,\{4\})$$
$$= R(\{1\}) \sum_{Z\in\Phi^C(1)} R_C(2,Z) = R(1)[R_C(2,\emptyset) + R_C(2,\{3\})]$$
$$= p^3q[p^8 + 4p^7q + 2p^6q^2 + p^7q + p^6q^2]$$
$$= p^{11}q + 5p^{10}q^2 + 3p^9q^3$$
$$R_C(3,\{13\}) = R_C(3,\{24\})$$

$$= R(\{13\}) \sum_{Z \in \Phi^C(13)} R_C(2,Z) = R(\{13\})[R_C(2,\emptyset)]$$
$$= p^2q^2[p^8 + 8p^7q + 8p^6q^2] = p^6q^2$$
$$= p^{10}q^2 + 8p^9q^3 + 8p^8q^4$$

Then,

$$R_C(3) = \sum_{X \in \Theta^C} R_C(3,X) = R_C(3,\emptyset) + 4R_C(3,\{1\}) + 2R_C(3,\{13\})$$
$$= p^{12} + 8p^{11}q + 8p^{10}q^2 +$$
$$+4p^{11}q + 20p^{10}q^2 + 12p^9q^3$$
$$+2p^{10}q^2 + 16p^9q^3 + 16p^8q^4$$
$$R_C(3) = p^{12} + 12p^{11}q + 30p^{10}q^2 + 28p^9q^3 + 16p^8q^4$$

For non-identical state, the reliability of component *i,j* is $p_{i,j}$, and then

$$R_C(1,\emptyset) = R(\emptyset) = \prod_{i=1}^{4} p_{i,j} = p_{1,1} \times p_{2,1} \times p_{3,1} \times p_{4,1}$$
$$R_C(1,\{1\}) = p_{2,1} \times p_{3,1} \times p_{4,1} \times q_{1.1} \qquad R_C(1,\{2\}) = p_{1,1} \times p_{3,1} \times p_{4,1} \times q_{2,1}$$
$$R_C(1,\{3\}) = p_{1,1} \times p_{2,1} \times p_{4,1} \times q_{3,1} \qquad R_C(1,\{4\}) = p_{1,1} \times p_{2,1} \times p_{3,1} \times q_{4,1}$$
$$R(1) = p_{2,1} \times p_{3,1} \times p_{4,1} \times q_{1.1} + p_{1,1} \times p_{3,1} \times p_{4,1} \times q_{2,1} + p_{1,1} \times p_{2,1} \times p_{4,1} \times q_{3,1} + p_{1,1} \times p_{2,1} \times p_{3,1} \times q_{4,1}$$
$$R(1,\{13\}) = R(\{13\}) = p_{2,1} \times p_{4,1} \times q_{1,1} \times q_{3,1}$$
$$R(1,\{24\}) = R(\{24\}) = p_{1,1} \times p_{3,1} \times q_{2,1} \times q_{4,1}$$

and then proceed solution.

4. Evaluation of the proposed algorithm

(Lin and Zuo 2000) introduced a recursive algorithm that required a computing complexity $O\left(\left[2^{r(s-1)}m^s + (n-s)m\right](r+1)r^{s(m-r+1)}\right)$, and then (Akiba *et al*. 2001) proposed more effective recursive algorithm with a computing complexity $O(mk^{rn})$, when $k = r = s = 2$, it is $O(m2^{2n})$.

In the proposed algorithm, $\Phi^{\mathrm{C}} \subseteq \Phi^{\mathrm{L}}$, and the number of the working states in $\Phi^{\mathrm{L(C)}}$ in any layer (circle) is less than $\sum_{j=0}^{\left[\frac{n+1}{2}\right]} \binom{n-j+1}{j}$ (as in Chang and Niu 1981), which is less than 2^{n-1}, as well as the number of $R(i,X)$, which applied for m layers (circles), i.e. the complexity of the proposed algorithm for $R_{L(C)}(m)$ is $O(m \times 2^{n-1} \times 2^{n-1}) = O(m2^{2n-2})$. (For the linear system if $n > m$, we can rotate the system 90^{o} degree to have $O(n2^{2m-2})$).

5. Conclusion

This paper introduced a recursive algorithm to compute the reliability function of the 2-within-consecutive-(2,2)-out-of-(*m*,*n*): F linear and circular system, in this context, the collections of all working states in the consecutive-2-out-of-*n*: F linear and circular system was helpful to compute the reliability of the system, where the complexity of the proposed algorithm is $\mathrm{O}(m2^{2n-2})$, which is less than the mentioned algorithms in the literature.

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