Review Of Stability Of Nonlinear Te Surface Waves Along The Boundary Of Gyromagnetic Or Gyrodielectric Media

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Abstract: *This paper is concerned with the stability characteristics of nonlinear surface waves propagating along a nonlinear dielectric cover and a linear substrate. These characteristics have been simulated numerically by using a perturbation method. The substrate may be a lateral antiferromagnetic nonmagnetic superlattices (LANS) or ferromagnetic or semiconductor media. LANS are linear frequency- dependent gyromagnetic media, described by an effective- medium theory. The growth rate of perturbation is computed by solving the dispersion equation of perturbation for LANS, we found that the nonlinear surface waves are unstable because their growth rate of perturbation δ is always real with the existence of the magnetic matter of permeability tensor μ^e. This means that the stability of nonlinear surface waves is magnetic fraction dependent. The spatial evolution of the steady state field amplitude is determined by using computer simulation method . The calculations show that with increasing the effective refractive index n_x at fixed saturation parameter. μ_p the field distribution is sharpened and concentrated in the nonlinear medium. For ferromagnetic or semiconductor substrate, we found that the nonlinear surface waves can be stable or unstable according to the frequency region.*

Key words:*nolinear waves, wave guides, dispersion magnetic super lattices, semiconductor, ferromagnet, growth rate, stability.*

1.Introduction

The propagation of electromagnetic waves along the boundary of gyromagnetic or gyrodielectric media is very interesting for both fundamental research and future applications[1-6] in devices such as isolators, switches, circulators, and signal processing devices . There are many methods to calculate the dispersion relation. Boardman et al. [2] introduced the boundary field method to give the wave index as a function of the field at the interface of the magnetized semiconductor. The question is whether these wave solutions are stable on propagation, of waves. To date this problem can not be considered as fully solved. Nevertheless, there is a number of approaches to the problem both using numerical simulations by Akhmediev et al. [6] and Moloney et al. [5]

and analytical methods which has been based on steady-state solutions to a nonlinear wave equation which contains an intensity dependent refractive index. Akhmediev et al. [6] had shown when the growth rate of perturbation of waves δ is real , the surface waves are unstable and when δ is imaginary, the waves are stable. This paper is concerned with the stability of nonlinear surface waves propagating along the boundary of linear (LANS) or ferromagnetic or semiconductor media , we solve this problem by using computer simulation method [7].

2.Basic Equations

The geometry is shown in fig. (1.1), where the nonlinear semi-infinite cladding contacts everywhere to linear, semi-infinite super lattice or ferromagnetic or semiconductor media at
y = 0 planar interface. The coordinate system is such that, the y axis is normal to the interface and the wave vector is directed along the x axis.

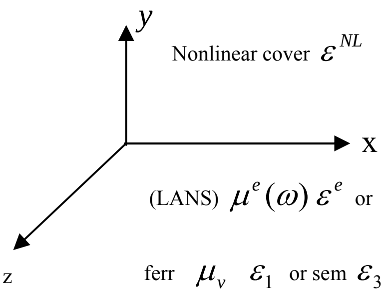

Fig.(1.1) Coordinate system guiding TE surface

Since the wave propagation is in x-direction then, the Maxwell equations for S-polarized wave (TE) are reduced to the following equation [6]

$$\nabla^{2}E+\varepsilon\left(y,|E|^{2}\right)E=0 \qquad (1)$$

The dielectric constant of the nonlinear medium , $y>0$ is

$$\varepsilon^{nl}=\varepsilon_{2}+\alpha|E|^{2} \qquad (2)$$

Assuming that the nonlinear medium is self-focusing, the solution of the wave equation which is polarized along the z-axis is

$$E_{z}(x,y)=\alpha^{1/2}\ A\,(x,y)e^{i(n_{x}x-\varpi t)} \qquad (3)$$

Where A (x,y) is a slowly varying field envelope, n_{x} is the effective refractive index by substituting Eq.(3) into Eq. (1), the equation for the slowly varying amplitude $A(x,y)$ is then [6] :

$$2i\,n_x \frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial y^2} - k_2^2(y)A + \frac{\alpha}{\alpha_0}|A|^2 A = 0 \tag{4}$$

where $k_2^2(y) = n_x^2 - \varepsilon_2$, is the decay constant of the nonlinear medium, ε_2 is the linear dielectric constant of the nonlinear medium, the fields are normalized by the factor $\alpha_0^{1/2}$ and $\alpha_\circ$ is the non-linearity coefficient., ω is the wave angular frequency and c is the light velocity in free space

To determine the stability criterion for NSWs, we numerically simulated the steady state solution of Eq. (4) with small perturbation as [6]:

$$A(x,y) = A_O(y) + \mu_p\, f(x,y) \tag{5}$$

Where $f(x,y)$ is a perturbation function of the steady state solution, μ_p is the saturation parameter. $A_O(y)$ is the steady-state solution of equation(4), so it can be written as:

$$A_O(y) = \begin{cases} 2^{\frac{1}{2}}\left(k_2^2 - \beta^2\right)^{1/2} e^{\beta y}\ ,\ y<0,\ \text{for LANS medium} \\ 2^{\frac{1}{2}}\left(k_2^2 - k_1^{\ 2}\right)^{1/2} e^{k_1 y}\ ,\ y<0,\ \textit{for ferromagnetic medium} \\ 2^{\frac{1}{2}}\left(k_2^2 - k_3^{\ 2}\right)^{1/2} e^{k_3\ y}\ ,\ y<0,\ \text{for semiconductor medium} \\ 2^{\frac{1}{2}} k_2 \sec h\left(k_2\left(y - y_\circ\right)\right), y>0, \text{for nonlinear medium} \end{cases} \tag{6}$$

β is the decay constant of the LANS, it is given by[3]:

$$\beta = \sqrt{(\mu_{xx}/\mu_{yy})n_x^2 - \varepsilon_\perp \mu_v} \tag{7a}$$

with

$$\mu_{xx} = \left[1/\left(f_1 + f_2\mu\right)\right]\left[\left(f_1^2 + f_2^2\right)\mu + f_1\ f_2\left(1 + \mu^2 - \mu_\perp^2\right)\right] \tag{7b}$$

$$\mu_{yy} = \mu/\left(f_1 + f_2\ \mu\right)$$

where the expressions of μ and $\mu_\perp$ are:

$$\mu = 1 + \omega_a\omega_m\left\{\left[\omega_r^2 - (\omega_0 - \omega)^2\right]^{-1} + \left[\omega_r^2 - (\omega_0 + \omega)^2\right]^{-1}\right\}, \tag{7c}$$

$$\mu_\perp = \omega_a\omega_m\left\{\left[\omega_r^2 - (\omega_0 - \omega)^2\right]^{-1} + \left[\omega_r^2 - (\omega_0 + \omega)^2\right]^{-1}\right\}$$

with $\omega_m = 4\,\pi\,\gamma\,m_0, \omega_a = \gamma\,H_a, \omega_0 = \gamma\,H_0$,

$$\omega_r = \gamma\sqrt{2\,H_a H_e + H_a^2}$$

where, H_a represents the an isotropy field, H_e the exchange field, and γ the gyromagnetic ratio, m_0 is the sublattice magnetization, ε_1 is the dielectric constant of the layers, ω_r is the resonance frequency.
The effective dielectric function of LANS is expressed [4] by:

$$\varepsilon_\perp = f_1\, \varepsilon_{11} + f_2\, \varepsilon_{22} \tag{7d}$$

k_1, k_3 are the decay constant of ferromagnetic and semiconductor medium respectively where

$k_1 = \sqrt{n_x^2 - \varepsilon_1 \mu_v}$, $k_3(y) = \sqrt{n_x^2 - \varepsilon_3}$,

and y_0 is the integration constant, that gives the position of the maximum in the E_Z in the upper nonlinear half space. At the interface between the two media $y = 0$, we assume the condition that the dielectric constant $\varepsilon_\perp > \varepsilon_2$, $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_3 > \varepsilon_2$.
We shall consider the z dependence of the perturbation function, so that the function can be written in the form [5]

$$f(x,y,z) = \frac{1}{2}\left[(u+v)e^{(\delta x + i r z)} + (u^* - v^*)e^{(\delta^* x - i r z)}\right], \tag{8}$$

where u and v are functions of y only. We take the case $r^2 = \delta^2$ for nonlinear medium.
Substituting Eq. (8) into Eq. (5), and Eq. (4) we get the set of differential equations in nonlinear medium and linear medium. u and v are the solutions of these equations .
For a surface wave the growth rate of perturbation is either real or imaginary, suppose that it is real thus by a bit of algebra we can obtain a dispersion relation for determining δ of the form [6]:

$$\left|p(1+i\xi) - 2i\xi t - 3pt^2 + 2t^3 - s(p-t)^2\right|^2 - (1-t^2)^2\left|p - 2t + \bar{s}\right|^2 = 0, \tag{9}$$

where $\xi = \xi'/k_2^2$, $\xi' = 2n_x\delta$,

and $p = (1+i\xi)^{1/2}$, $t = \tanh(k_2 k_0 y_\circ)$ which implies $0 < t < 1$

for LANS

$$\tanh(k_0 k_2 y_0) = \frac{-\beta\, \mu_{yy} + n_x\, \mu_{xy}}{\mu_{yy}\, \mu_v\, k_2}, \tag{10a}$$

The above equation can be easily rewritten as:

$$y_O = \frac{1}{2k_2k_0} \ln \frac{\mu_{yy}\mu_v k_2 - \beta\mu_{yy} + n_x\mu_{xy}}{\mu_{yy}\mu_v k_2 + \beta\mu_{yy} - n_x\mu_{xy}} \; , \; s = \left(\frac{\beta^2}{k_2^2} - i\xi\right)^{1/2}$$

for ferromagnetic substrate

$$\tanh(k_0 k_2 y_0) = \frac{-k_1\,\mu_{xx} + n_x\,\mu_{xy}}{\mu_{xx}\,\mu_v\,k_2}\,, \tag{10b}$$

where

$$y_O = \frac{1}{2k_0 k_2}\ln\frac{\mu_{xx}\,\mu_v\,k_2 - k_1\mu_{xx} + n_x\mu_{xy}}{\mu_{xx}\,\mu_v\,k_2 + k_1\mu_{xx} - n_x\mu_{xy}}\,, \qquad s = \left(\frac{k_1^2}{k_2^2} - i\,\xi\right)^{1/2}$$

and $\mu_v = \mu_{xx} + (\mu_{xy}^2 / \mu_{xx})$ is called the Voigt permeability [8] .

$$\mu_{xx} = \mu_B\left(\frac{\omega_0(\omega_0 + \omega_m) - \omega^2}{\omega_0^2 - \omega^2}\right), \quad \mu_{xy} = i\,\mu_B\,\frac{\omega\omega_m}{\omega_0^2 - \omega^2}, \quad \omega_0 = \gamma\mu_0 H_0, \omega_m = \gamma\mu_0 M_0$$

where H_0 is the DC applied magnetic field , M_0 is the DC magnetization , μ_0 is the magnetic permeability of free space, μ_B is the background permeability and γ is the gyromagnetic ratio.

for semiconductor

$$\tanh(k_0 k_2 y_0) = \frac{k_3}{k_2}\,, \tag{10c}$$

$$s = \left(\frac{k_3^2}{k_2^2} - i\,\xi\right)^{1/2}, y_O = \frac{1}{2k_0 k_2}\ln\frac{k_2 + k_3}{k_2 - k_3}$$

and

$$\varepsilon_3 = \varepsilon_\infty\left[1 - \frac{(\omega + i\upsilon)\omega_p^2}{\omega((\omega)^2 - \omega_c^2)}\right] \;, \omega_p^2 = \frac{ne^2}{\varepsilon_\infty \varepsilon_0 m^*} \quad , \omega_c = \frac{eB_0}{m^*}\, and \quad B_0 = \mu_0 H_0$$

where m^* is the effective mass of the electron, H_0 is the DC applied magnetic field , ω_p is the plasma frequency , ω_c is the cyclotron frequency and ε_∞ is the background dielectric constant as the wave frequency $\omega \to \infty$.

Eq. (9) may be solved analytically, one obtain [5] $\xi_r^2 = 0.533(1 - 2t)$ (11)

when $t < \frac{1}{2}$, $\xi_r^2 > 0 \Rightarrow \xi_r$ is real, the growth rate δ is related to ξ_r by [4] $\delta = \frac{\xi_r\,k_2^2}{2\,n_x}$ which cause the NSW to be unstable. When

$t > \dfrac{1}{2}, \xi_r^2 < 0 \Rightarrow \xi_r$ is imaginary where δ becomes imaginary and NSW is stable. At $t = \dfrac{1}{2}$, n_x in this case is the critical refractive index. The evolution of the perturbed field amplitude $A(y)$ at the propagation distance x, is calculated through applying the boundary conditions at y = 0 as [5]:

1) $E_{z_l} = E_{z_{Nl}}$,it is found by substituting Eq.(5)and Eq(6) into Eq(3)

2) $B_{x_l} = B_{x_{Nl}}$, B is the magnetic induction of the system [4] , (12)

For **LANS**

$$B_{x_{Nl}} = \frac{\mu_{x_{Nl}}}{\omega \mu_\circ} \frac{\partial E_{z_{Nl}}}{\partial y}, \quad B_{x_l} = \mu_{xx} H_{x_l} + i \mu_{xy} H_{y_l} \quad \text{(13a)}$$

where

$$H_{x_l} = \frac{-\beta\mu_{yy} + n_x \mu_{xy}}{\omega \mu_\circ \mu_v \mu_{yy}} E_z, \quad H_{y_l} = \frac{n_x \mu_{yy} - \beta\mu_{xy}}{\omega \mu_\circ \mu_v \mu_{yy}} E_z$$

and for **ferromagnetic substrate**

$$H_{x_l} = \frac{-k_1 \mu_{ee} + n_x \mu_{xy}}{\omega \mu_\circ \mu_v \mu_{ee}} E_z, \quad H_{y_l} = \frac{n_x \mu_{ee} - k_1 \mu_{xy}}{\omega \mu_\circ \mu_v \mu_{ee}} E_z \quad \text{(13b)}$$

also for **semiconductor substrate** the boundary condition (2) is:

$$\frac{\partial E_{znl}}{\partial y} = \frac{\partial E_{zl}}{\partial y} \quad \text{(13c)}$$

Since the wave function u vanishes at the boundary, say y =10 then

3) $u_{N_l} = 0$ at $y = 10$ 4) $u_l = 0$ at $y = -10$

3.COMPUTER SIMULATION AND DISCUSSION:

The numerical calculations are presented for the simulation of the stability equation (5) of the proposed structure. It consists of nonlinear dielectric cover and a lateral *FeF_2 /ZnF_2* superlattice LANS or ferromagnetic or semiconductor. *FeF_2* is ferrous fluoride which is antiferromagnetic film and *ZnF_2* is zinc fluoride which is nonmagnetic film.

The parameters of LANS are [4] as follows: $\varepsilon_{11} = 5.5$ for antiferromagnetic layers and $\varepsilon_{22} = 8$ for nonmagnetic layers For ferromagnetic, the parameters are [8] :

The applied field, $\mu_0 H_0 = 5600G$, $\mu_0 M_0 = 1750G$, $\gamma = 1.97 \times 10^7\ rad/\sec.G$ and $\varepsilon_1 = 16$. For a semiconductor substrate, the parameters are [9] : $\omega_p = 1.8x10^{15}\ rad/s$, $\varepsilon_\infty = 15.68$

For Lans Substrate

The calculations show that with increasing n_x at fixed μ_P, the field distribution is sharpened and shifted in nonlinear medium as shown in Figs (1.4a-b-c) at $x = 3$. This conclusion is different from that obtained by. Akhmediev et al. [6] in which the linear medium is nonmagnetic, δ is real for t $< \frac{1}{2}$ at which the surface waves are unstable and δ is imaginary for t $> \frac{1}{2}$ at which the waves are stable.

For Ferromagnetic Substrate

The calculations show that with increasing values of n_x (3, 3.5, 4) at fixed μ_P, the spatial evolution of perturbations (δ = 0.2, δ = 0.43*I δ = 0.73*I) respectively leads to a shift of the field amplitude $A_l(x, y)$ into nonlinear medium with the subsequent excitation of the nonlinear surface waves of the stable branch as shown in Fig.(2.1a-b-c). Fig.(2.2a-b) shows the variation of the field distribution with the wave frequency at the propagation length $x = 3$ and n_x = 4. It displays that with increasing the wave frequency from f = 20.5x10^9 Hz, to f = 21.5x10^9 Hz the growth rate turns from δ = 0.925 (unstable) into , δ = 0.586*I (stable). This means that increasing of wave frequency causes excitation of stable surface waves. These results are different from those obtained for (LANS) where the magnetic fraction f_1 causes the growth rate δ to be always real and the waves are always unstable.

For Semiconductor Substrate

For increasing values of n_x Fig.(3.1a-b-c) display the spatial evolution of steady state field amplitude $A_l(x, y)$ as a function of the refractive index n_x. We found that at n_x = 6, the perturbed waves are unstable where the growth rate of perturbation δ is real (δ = 1.67). For increasing values of n_x to (7and 8) the growth rate δ becomes imaginary of values (0.74*I, 1.7*I) respectively where the waves become stable and shifted to the nonlinear medium, with the subsequent excitation of the nonlinear stable surface waves of higher energy. Fig.(3.2a-b) show the variation of the field distribution at the propagation distance (x = 3) and n_x = 8 with the cyclotron frequency ω_c. It displays that with increasing the cyclotron frequency to the values ($0.6\omega_p$, 1.2 ω_p) at fixed n_x, the growth rate δ becomes (1.87, 2.1*I) respectively as shown by Eq.(9) and the field distribution is sharpened where the waves turn from unstable to stable waves and concentrated in the nonlinear medium. This means that the stability of the waves is affected by the cyclotron frequency.

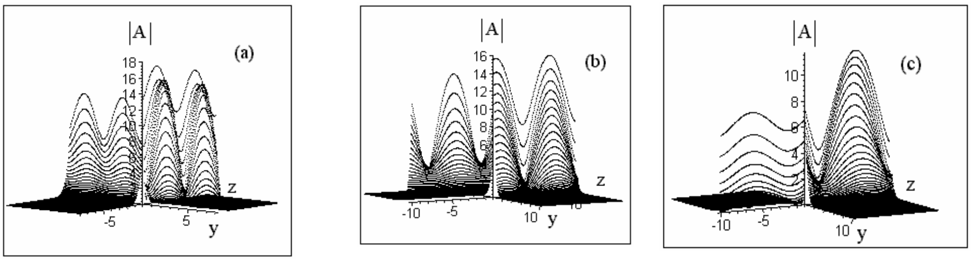

Fig.(1.4a-b-c).The field distribution A(y,z) for(a) $n_x = 5.2$, $\delta = 1.493$ (b)) $n_x = 4$, $\delta = .9554$, (c) $n_x = 3.5$, $\delta = .725$ for $\mu_p = 0.3$, $f_l = 0.8$, $\omega = 997.3\text{x}10^{10}$ rad / sec , x = 3.

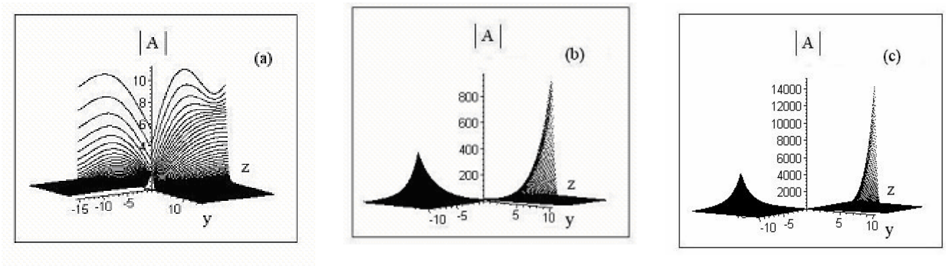

Fig.(2.1a-b-c).The field distribution of the nonlinear surface waves A(y,z) for (a) $n_x = 3$, growth rate $\delta = 0.2$ (b) $n_x = 3.5$, $\delta = 0.43*I$ and (c) $n_x = 4$, $\delta = 0.73*I$ for $\mu_p = 0.3$, $f = 21\text{x}10^9$ Hz, and propagation distance x = 3.

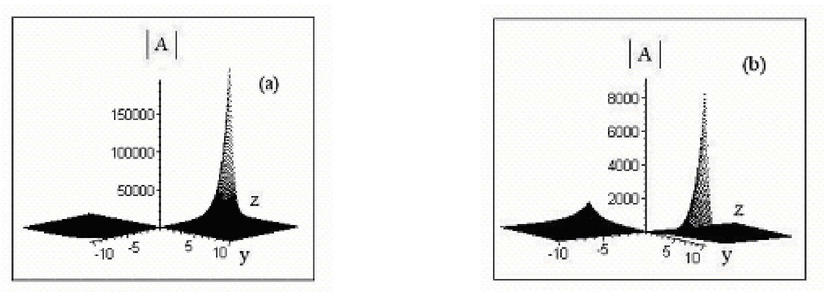

Fig.(2.2a-b).The field distribution of the nonlinear surface waves A(y,z) for (a) f = 20.5×10^9 Hz, growth rate δ = 0.925 and (b) f = 21.5×10^9 Hz, δ = 0.586*I for n_x = 4, $\mu_p = 0.3$, and propagation distance x = 3.

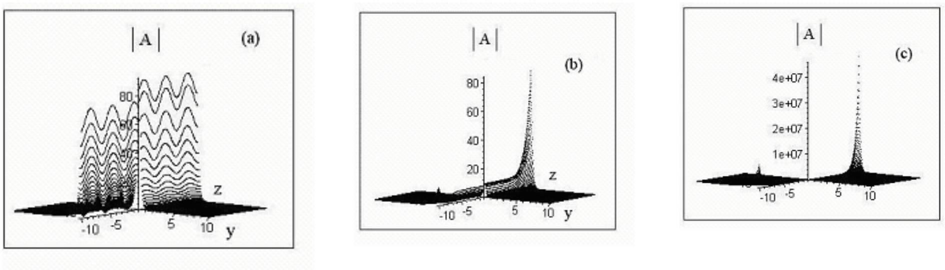

Fig.(3.1a-b-c).The field distribution of the nonlinear surface waves A(y,z) for (a) n_x = 6, growth rate δ =1.67 (b) n_x = 7, δ =.74*I and (c) n_x = 8, δ =1.7*I for $\mu_p = 0.3$, f = 21×10^6 Hz, $\omega_c / \omega_p = 0.9$, $\varepsilon_3 = 35$ and propagation distance x = 3.

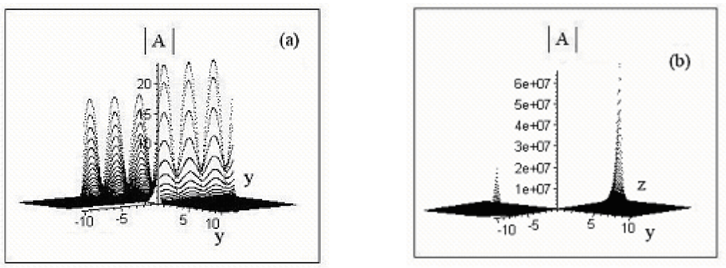

Fig.(3.2a-b).The field distribution of the nonlinear surface waves A(y,z) for (a) $\omega_c / \omega_p = 0.6$, $\varepsilon_3 = 59.2$, δ =1.87, (b) $\omega_c / \omega_p = 1.2$, δ =2.1*I, $\varepsilon_3 = 26.5$ for n_x = 8, $\mu_p = 0.3$, f = 21×10^6 Hz, and x = 3.

4. **Conclusions**

For LANS, the stability of surface waves is magnetic fraction dependent but for Ferromagnetic, the stability of surface waves is frequency dependent and for Semiconductor, the stability of surface waves is affected by cyclotron frequency. We believe that this work represents an important research in investigation of stability of nonlinear waves in gyromagnetic and gyrodielectric media.

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