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Nonlinear TE Surface Waves in Linear-Nonlinear Nonmagnetic LHM Structure

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Abstract*: In this paper, we reintroduce the design of nonlinear nonmagnetic left handed material (LHM). We derived the dispersion relation for linear-nonlinear nonmagnetic LHM substrate. Then, we numerically solved the dispersion relation of the nonlinear TE surface waves in the structure and the power flow. We found the wave effective refractive index is slightly dependent on the nonlinearity of the structure and the power flow is higher for higher frequencies, whereas the power loss is lower for higher frequencies.*

Introduction

In 1968, Veselago [1] suggested, in his pioneer paper, that electromagnetic propagation in an isotropic medium with negative dielectric permittivity $\varepsilon(\omega)<0$ and negative permeability $\mu(\omega)<0$ can exhibit unusual properties. In such material (LHM) the electric field vector $\vec{E}$, the magnetic field vector $\vec{H}$, and the wave vector $\vec{k}$ form a left hand orthogonal set. Recently (2001), a group of researchers from the University of San Diego were able to synthesize artificial dielectric medium (metamaterials) and they were able to demonstrate that these materials exhibit both negative dielectric permittivity and negative magnetic permeability simultaneously over a certain rang of frequencies [2]. These recent demonstrations of the existence of the LHM opened the door wide open to unique possibility to design a novel type of devices based on electromagnetic wave propagation in these materials but in non-conventional way. Pendry and others [3-5] demonstrated that a slab of lossless LHM can provide a perfect image of a point source. A slab of the LHM can serve as a perfect lens sometimes called superlenses, since the resolution of these lenses do not depend on the wavelength of the illuminating wave. Shadrivov et al. [6] have studied the guided modes in a negative-refractive-index (LHM) slab waveguide. They found some peculiar properties such as the absence of the fundamental modes, mode double degeneracy, and sign varying energy flux.

Recently, Shadrivov et al. [7, 8] proposed nonlinear LHM structure, and Podolskiy and Narimanov [9] proposed nonmagnetic linear LHM. Also, Hamada, Shabat, and Jäger [10] have investigated the propagation

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characteristics of the nonlinear TM surface waves in a left-handed material structure.

In this paper, we are going to propose a new lossless nonlinear and nonmagnetic LHM and study the electromagnetic waves propagation in linear-nonlinear nonmagnetic LHM structure in GHz range of frequencies.

This paper is organized as follows, section II is devoted to the nonlinear nonmagnetic LHM structure. In section III, we derive the dispersion relation of the surface waves in linear-nonlinear and nonmagnetic LHM structure and the power flow. In section IV, we discuss the results. Section V, is solely devoted to the conclusion.

Nonlinear Nonmagnetic LHM

Left handed materials are composite materials do not exist in nature. Researchers have to introduce arrays of split ring resonators and wires in the host material to change their magnetic and electric properties. The function of the split ring resonator SRR is to make the effective value of the magnetic permeability negative over a certain range of frequencies, and that of the wires in the structure is to make the effective permittivity of the composite material negative over the same range of frequencies [2]. Shadrivov et al. [6] had proposed the structure and the properties of the nonlinear LHM. They had used a two dimensional composite structure consisting of square lattice of periodic arrays of conducting wires and SRRs impeded in nonlinear dielectric material to induce both nonlinear effective electric permittivity and nonlinear magnetic permeability. Also, they assumed the unit cell size d to be smaller than the wavelength of the propagating EM field [2]. This simultaneous nonlinearity in electric and magnetic properties of the LHM make the structure very hard to deal with, especially if we are going to study the electromagnetic waves propagation characteristics when this material is used with other class of materials. Recently, Podolskiy and Narimanov [9] had proposed the use of linear nonmagnetic left handed material with $\mu = 1$, which is a helpful breakthrough that enables us to solve the dispersion relation for any structure. The effective nonlinear dielectric permittivity is induced by an array of conducting wires hosted in nonlinear dielectric medium. The geometry and the physical characteristic of the conducting wires are d is the wire spacing, r is wire radius, and c is the speed of light. Assuming no losses, the effective nonlinear dielectric permittivity is [6-8]

$$\varepsilon_{eff}^{NL}\left(|E|^2\right) = \varepsilon_D\left(|E|^2\right) - \frac{\omega_p^2}{\omega^2}, \qquad (1)$$

where $\omega_p \approx (c/d)\left[2\pi/\ln(d/r)\right]^{1/2}$ is the effective plasma frequency, and $\varepsilon_D\left(|E|^2\right)$ is Kerr nonlinearity of the dielectric in the composite material

$$\varepsilon_D\left(|E|^2\right) = \varepsilon_{D0} + \alpha |E|^2 / E_c^2, \tag{2}$$

where E_c is the characteristic electric field, ε_{D0} is the linear dielectric constant, and $\alpha = \pm 1$ stands for the focusing or defocusing nonlinearity, respectively [6-8]. Notice the second term of equation (1) is in a full agreement with the earlier result obtained by Pendry et al [15].

It is worthwhile to notice that the nonlinear nonmagnetic LHM behaves like metals but with better performance and this is because the host material is nonlinear dielectric material and the electric permittivity can be controlled by controlling the physical properties of wires. This gives the nonlinear nonmagnetic left handed material advantage over the metals and nonlinear dielectric materials.

Theory

a. **The Dispersion Relation**

Figure 1, shows the geometry and coordinates of the structure under investigation. The structure contains the nonlinear nonmagnetic left-handed material cover $(z > 0)$ and the linear left handed material substrate $(z < 0)$. We briefly outline the derivation of the dispersion relation for TE surface waves in the structure. Here, we consider TE polarized wave propagating along the z direction with angular frequency ω and effective wave number β. The electromagnetic field components are:

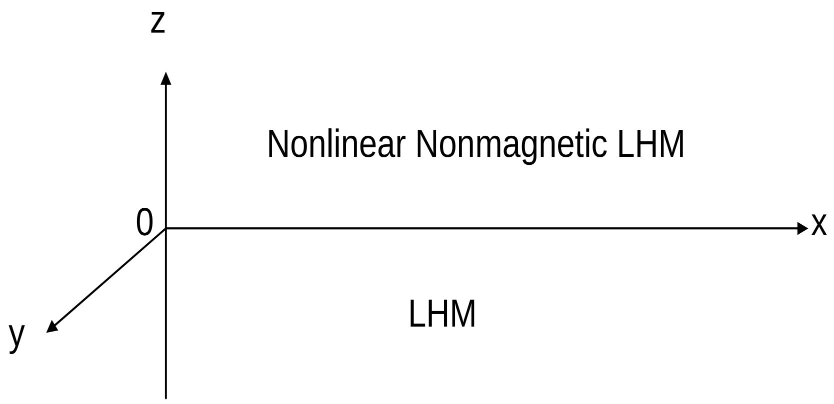

Figure (1): The geometry of the problem, nonlinear nonmagnetic left-handed material, the cover, and the LHM, the substrate.

$$E = \left[0, E_y, 0\right]\exp\left(ik_0\left(\beta z - ct\right)\right) \tag{3}$$

$$H = \left[H_x, 0, H_z\right]\exp\left(ik_0\left(\beta z - ct\right)\right) \tag{4}$$

where $k_0 = \omega / c$.

Substitution of equations (3) and (4) into Maxwell's equation yields the following nonlinear differential equation to satisfy E_{y1} in the nonlinear nonmagnetic LHM cover[14]:

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(\beta^2 - \varepsilon_{D0} - \alpha_1 E_y^2\right)E_y = 0, \qquad (5)$$

Where $\alpha_1 = \alpha / E_c$. The solution of the nonlinear differential equation (5) has the following form [12,13],

$$E_{y1} = k_1 / k_0 \left(2/\alpha_1\right)^{1/2} \sec h\left(k_1\left(z - z_0\right)\right), \qquad (6)$$

where $k_1 = k_0\left(\beta^2 - \varepsilon_{D0}\right)^{1/2}$ is the wave number in the nonlinear nonmagnetic LHM region, and z_0 is the integration constant where the maximum of the electric flied lies in the nonlinear region.

Also, the substitution of equations (3) and (4) into Maxwell's equation yields the following linear differential equation to satisfy E_{y2} in the linear LHM substrate:

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(\beta^2 - \varepsilon_{eff}^L \mu_{eff}^L\right)E_y = 0, \qquad (7)$$

where ε_{eff}^L and μ_{eff}^L are the effective permittivity and permeability of the linear left handed material respectively, and they are given by,

$$\varepsilon_{eff}^L = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (8)$$

$$\mu_{eff}^L = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}. \qquad (9)$$

Where ω_p is the plasma frequency, ω_0 is the resonance frequency of the wire, and F is the structure factor [14, 15].

The solution of equation (7) is:

$$E_{y2} = A\exp\left(k_2 z\right), \qquad (10)$$

Where $k_2 = k_0\left(\beta^2 - \varepsilon_{eff}^L \mu_{eff}^L\right)$ is the wave number in the linear LHM substrate, and A is the wave amplitude at the interface.

Using equations (6) and (10) and applying the continuity conditions to obtain the dispersion relation, that is

$$\beta^2 = \left[\frac{\mu_{eff}^{L\,2}}{\left(\mu_{eff}^{L\,2} - 1\right)}\right]\left[\frac{\varepsilon_{eff}^L}{\mu_{eff}^L} - \varepsilon_D^{'} - \frac{A^2\alpha_1}{2}\right] \qquad (11)$$

Where $\varepsilon_D^{'}$ is $\varepsilon_{D0} - \dfrac{\omega_P^2}{\omega^2}$. Thus the amplitude at the interface can be obtained as:

$$A^2 = (2/\alpha_1)\left[(\beta^2 - \varepsilon_{D0})\mu_{eff}^{L\ 2} - (\beta^2 - \varepsilon_{eff}^L \mu_{eff}^L)\right] / \mu_{eff}^L . \tag{12}$$

b. **Power Flow**

The power flux of the TE surface waves along the direction of propagation can be found by integrating the Poynting vector

$$P = \frac{1}{2}\int (E \times H^*) dz = \frac{1}{2}\int E_y H_x dz \equiv P_{NLHM} + P_{LHM} , \tag{13}$$

where P_{NLHM} and P_{LHM} stand for the power flux in the nonlinear nonmagnetic LHM and linear LHM respectively.

$$P_{NLHM} = 0.5(\beta k_1^2 / k_0\ \omega\mu_0)(2/\alpha_1)(1 + k_2 / k_1 \mu_{eff}^L), \tag{14}$$

and,

$$P_{LHM} = 0.25(\beta k_0 / k_2\ \omega\mu_0 \mu_{eff}^L) A^2 , \tag{15}$$

where A is given by equation (12).

Numerical Results and Discussion

The dispersion relation, equation (11), is solved numerically to calculate the propagation characteristics: the effective refractive index, the effective loss factor, and the power flux for the structure under investigation. The parameters of the linear LHM and the nonlinear nonmagnetic LHM are adjusted so that the parameters $\varepsilon_{eff}^L, \mu_{eff}^L$, and ε_{eff}^{NL} are negative in the same frequency range. The numerical calculations were carried out for the following numerical values: $\omega_p = 2\pi \times 10^9 \, rad/s,$ $\alpha = 1$ $\varepsilon_{D0} = 2.5,\ F = 0.56,\ E_c = 0.2\frac{V}{m},\ \ \omega_0 = 10^{10} \, rad/s$ and u is the interface nonlinearity equals $A^2\alpha_1/2$.

In figure 2-a, we plot the effective refractive index, or the wave index, of the structure versus the frequency range. The slope of the dispersion curves represents the group velocity [16] which is slightly sensitive to the optical power.

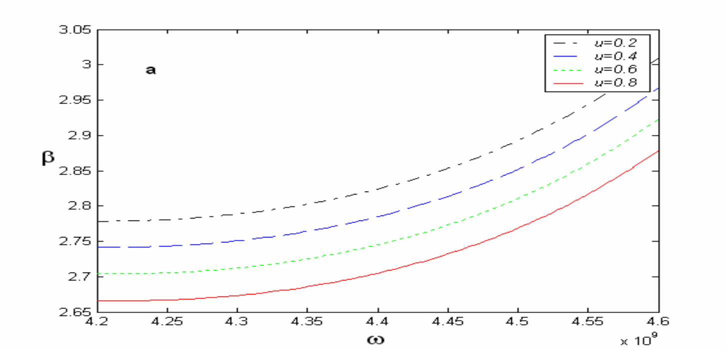

Figure 2-a: Dispersion curves of the TE-polarized surface waves for different values of the interface nonlinearity.

But in figure 2-b the effective nonlinear wave index increases with the optical power and frequency increase. As we can notice that the dispersion curves have a positive slope which means the power flow in the positive z-direction [16], so it behaves like RHMs. It is worthwhile to notice that the group velocity almost constant despite the change in the optical power increases and operating frequency. In figure 3-a, we plot the normalized power flow P/P_0, where P_0 *is* $\frac{1}{2\alpha_1\varepsilon_0\omega}$, versus the operating frequency and the effective nonlinear wave index. The power flow decreases with frequency increase up to certain limit then the relation is reversed, as in figure 3.a. In figure 3.b, the power loss decreases with frequency increase up to certain frequency then it changes its course but with slower rate than before. Also, figure 3.b, displays some kind of bistability which is very important in designing some devices as isolators, switches and circulators. In figure 3-c, we plot the normalized power versus the interface nonlinearity at several frequency values in the range, the power flow decreases with the optical power increase.

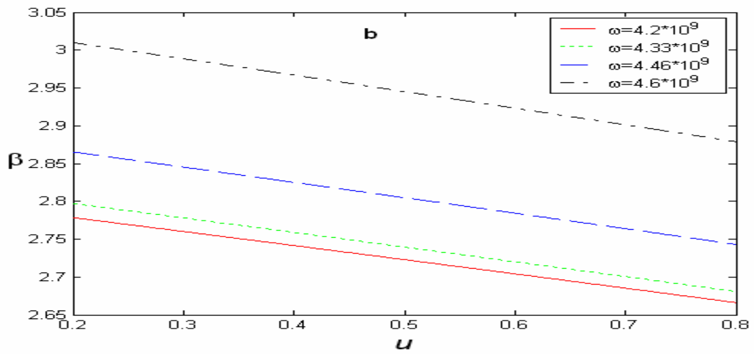

Figure 2-b: The effective wave index versus the optical power at several frequency values in the range.

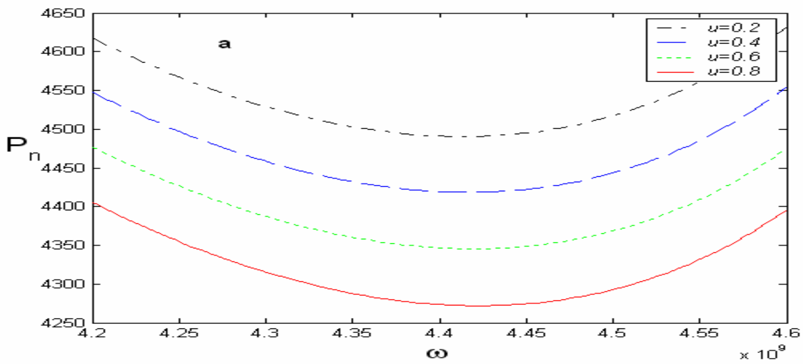

Figure (3.a): The normalized power versus the frequency for several values of interface nonlinearity.

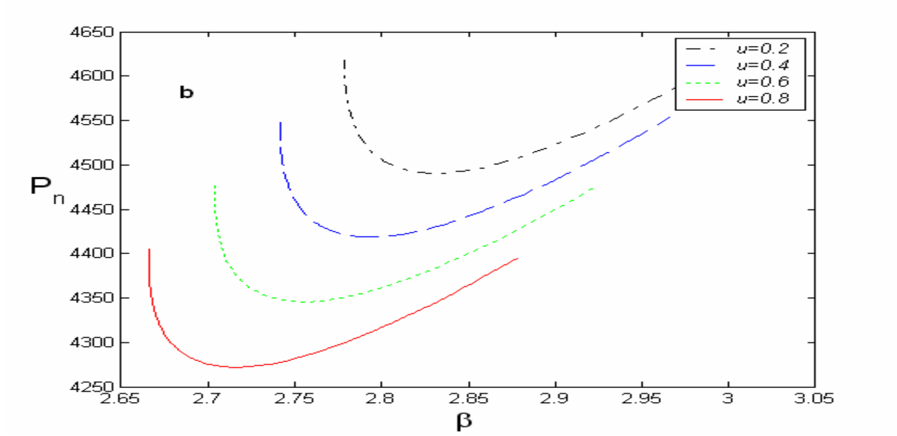

Figure (3.b): The normalized power versus the effective wave index for several values of interface nonlinearity.

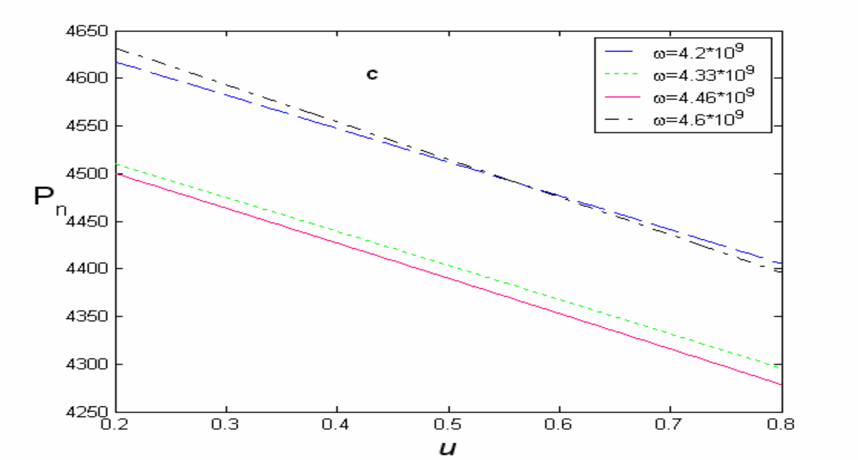

Figure 3-c: The normalized power versus the interface nonlinearity at several frequency values in the range.

Conclusion

We have reintroduced the design of the nonlinear nonmagnetic left handed material. In which this material behaves like a metal with a better advantage over metals since we can control the physical characteristics. The dispersion relation for linear-nonlinear nonmagnetic LHM structure were theoretically driven and then numerically solved to study the nonlinear TE surface waves in the structure and the power flow. We found the wave effective refractive index is increasing with frequency increase. The power flow is changing by changing the operating frequency of the allowed range. These promising characteristics could be used in future in designing some microwave-optoelectronic devices.

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